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QED effects are known to occur in a strong laser pulse interaction with a counterpropagating electron beam, among these effects being electron-positron pair creation. We discuss the range of laser pulse intensities of $J \geq 5\times 10^{22}$ W/cm^2 combined with electron beam energies of tens of GeV. In this regime multiple pairs may be generated from a single beam electron, some of the newborn particles being capable of further pair production. Radiation backreaction prevents avalanche development and limits pair creation. The system of integro-differential kinetic equations for electrons, positrons and γ photons is derived and solved numerically.

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Introduction.—The effects of quantum electrodynamics (QED) may occur in a strong laser pulse interaction with a counterpropagating electron beam. In a well-known experiment [1] these effects were weak and barely observable. If the laser pulse intensity is increased up to $J \geq 5\times 10^{22}$ W/cm^2 the QED effects control the laser-beam interaction and result in multiple pair production from a single beam electron.

QED-strong fields.—In QED an electric field, E, should be treated as strong if it exceeds the Schwinger limit: $E \geq E_S = m_e c^2/(|e|\lambda\!\!\!^{-}_C)$ (see Ref. [2]). Such field is potentially capable of separating a virtual electron-positron pair providing an energy, which exceeds the electron rest mass energy, $m_e c^2$, to a charge, $e = -|e|$, over an acceleration length as small as the Compton wavelength, $\lambda\!\!\!^{-}_C = \hbar/(m_e c) \approx 3.9\times 10^{-11}$ cm. Typical effects in QED-strong fields are electron-positron pair creation from high-energy photons, high-energy photon emission from electrons or positrons and the cascade development (see Refs. [3,4]) resulting from the first two processes.

Less typical is direct pair separation from vacuum. This effect may only be significant if the field invariants as defined in Ref. [5], $F_1 = (\mathbf{B}\cdot\mathbf{E})$, $F_2 = E^2 - B^2$, are large enough. Here the case of *weak* field invariants is considered: $|F_{1,2}| \ll E_S^2$, and any corrections of the order of $F_{1,2}/E_S^2$ are neglected (see Ref. [6] about such corrections). Below, the term "strong field" is only applied to the field experienced by a charged particle.

QED-strong laser fields.—QED-strong fields may be created in the focus of an ultrabright laser. Consider QED effects in a relativistically strong pulsed field [3]:

$$|\mathbf{a}| \gg 1, \qquad \mathbf{a} = \frac{e\mathbf{A}}{m_e c^2}, \tag{1}$$

$\mathbf{A}$ being the vector potential of the wave. In the laboratory frame of reference the electric field is not QED strong for achieved laser intensities, $J \sim 10^{22}$ W/cm^2 [7], and even for the $J \sim 10^{25}$ W/cm^2 intensity projected [8]. Moreover, both field invariants vanish for 1D waves, reducing the probability of direct pair creation from vacuum by virtue of the laser field's proximity to a 1D wave.

Nonetheless, a counterpropagating particle in a 1D wave, $\mathbf{a}(\xi)$, $\xi = \omega t - (\mathbf{k}\cdot\mathbf{x})$, may experience a QED-strong field, $E_0 = |d\mathbf{A}/d\xi|\omega(\mathcal{E} - p_\|)/c$, because the laser frequency, $\omega = c/\lambda\!\!\!^{-}$, is Doppler upshifted in the frame of reference comoving with the electron. Herewith the electron dimensionless energy, $\mathcal{E}$, and its momentum are related to $m_e c^2$, and $m_e c$ correspondingly, and subscript $\|$ herewith denotes the vector projection on the direction of the wave propagation. The Lorentz-transformed field exceeds the Schwinger limit, if $\chi \sim E_0/E_S \geq 1$. Numerical values of the parameter, χ, may be expressed in terms of the local instantaneous intensity of the laser wave, J:

$$\chi = \frac{3}{2}\frac{\lambda\!\!\!^{-}_C}{\lambda\!\!\!^{-}}(\mathcal{E} - p_\|)\left|\frac{d\mathbf{a}}{d\xi}\right| \approx \frac{(\mathcal{E} - p_\|)}{1.4\times 10^3}\sqrt{\frac{J}{10^{23}[\mathrm{W/cm^2}]}}. \tag{2}$$

In the experiment [1] an electron beam of energy ≈ 46.6 GeV interacted with a counterpropagating terawatt laser pulse of intensity $J \sim 10^{18}$ W/cm^2 ($|\mathbf{a}| \leq 1$). A value of $\chi \approx 0.4$ had been achieved. An increase in the laser field intensity up to $\sim 5\times 10^{22}$ W/cm^2 ($|\mathbf{a}| \approx 110$) with the use of the same electron beam, would allow us to reach a regime of multiple pair creation at $\chi \approx 90$.

Radiation backreaction.—The creation of pairs in QED-strong fields is a particular form of the radiation losses from charged particles. At high χ an intermediate stage in the pair creation process is the emanation of a high-energy photon by a charged particle: $e \to \gamma$, e (in contrast with $\chi \leq 1$ case, in which the "equivalent" photons from the electron Coulomb potential mostly contribute to the pair creation—see Ref. [4] and the papers cited therein). This photon is then absorbed in the strong field, generating an electron-positron pair: $\gamma \rightarrow e, p$.

Although the energy-momentum *gain* from the strong laser field is crucial in the course of both emission and pair creation, still a way to quantify the irreversible radiation *losses* may be found. Specifically, in the 1D wave field the transfer of energy, $\Delta\mathcal{E}$, from the wave to a particle may be interpreted as the absorption of some number of photons, n: $\Delta\mathcal{E} = n\hbar\omega/(m_e c^2)$. Accordingly, the momentum from the absorbed photons is added to the parallel momentum of the particle: $\Delta p_{\parallel} = n\hbar k/(m_e c) = n\hbar\omega/(m_e c^2)$. So, both energy and parallel momentum are not conserved; however, their difference is: $\Delta(\mathcal{E} - p_{\parallel}) = 0$. To get the Lorentz-invariant formulation, we introduce the four-vector of the particle momentum, $p = (\mathcal{E}, \mathbf{p})$, and the wave four-vector, $k = (\frac{\omega}{c}, \mathbf{k})$ for the 1D wave field. Their four-dot product, $(k \cdot p) = \omega(\mathcal{E} - p_{\parallel})/c$, is conserved in any particle interaction with the 1D wave field, including its motion, photon emission, pair creation, etc. The sum of this quantity, $\sum(k \cdot p_f)$, over all particles in the final (f) state is equal to that for the particles in the initial (i) state: $\sum(k \cdot p_f) = \sum(k \cdot p_i)$. Each term in this conserving sum is positive [we use the metric signature $(+, -, -, -)$]. Therefore, any contribution to this sum from a newborn particle exacts a contribution from its parent.

Regarding the high-energy electron beam interaction with the ultrastrong laser pulse, the initially high value of $\chi \approx 90$ ensures multiple pair creation. The radiation back-reaction, however, splits the initially high value of $(k \cdot p)$ between all newborn particles. The reduced values of $(k \cdot p)$ result in smaller values of $\chi \propto (k \cdot p)$. The cascade terminates, when all particles have $\chi \leq 1$ and become incapable of creating new pairs.

The radiation losses, thereby limit the cascading pair creation. Particularly, emission of softer γ photons may even be described within the radiation force approximation, which is traditionally used to account for the radiation backreaction (see Refs. [5,9–11]).

The discussed processes are described by the kinetic equations for the involved particles (electrons, positrons, γ photons). For a circularly polarized 1D wave of constant amplitude, the system of three 1D integro-differential kinetic equations is reducible to a large system of differential equations, which is solved here numerically.

Electrons in QED-strong fields.—The emission probability in the QED-strong 1D wave field may be found in Sections 40, 90, 101 in Ref. [12], as well as in Ref. [13]. However, to simulate highly dynamical effects in pulsed fields, one needs a reformulated emission probability, related to short time intervals [not $(-\infty, +\infty)$], which is rederived in Appendix A in Ref. [14] with careful attention to consistent problem formulation.

Again, the energy, $\hbar\omega'$, and momentum, $\hbar\mathbf{k}'$, of the emitted photon are normalized to $m_e c^2$ and $m_e c$. The four-dot product, $(k \cdot p)$, is the motional invariant for an electron and it is also conserved in the process of emission: $(k \cdot p_i) = (k \cdot k') + (k \cdot p_f)$. A subscript i or f denotes the electron in the initial (i) or final (f) state.

In the 1D wave field the emission probability may be conveniently related to the interval of the wave phase, $d\xi$, which should be taken along the electron trajectory. The interval of time, dt, and that of the electron proper time, $d\tau_e$, are related to $d\xi$ as follows: $d\tau_e = dt/\mathcal{E} = d\xi/[c(k \cdot p)]$. The phase volume element for the emitted photon is chosen in the form $d^2\mathbf{k}'_\perp d(k \cdot k')$. The emission probability, $dW_{fi}/[d\xi d(k \cdot k')]$, is integrated over $d^2\mathbf{k}'_\perp$; therefore, it is related to the element of the phase volume, $d(k \cdot k')$ (see detail in Appendix A in Ref. [14]):

$$\frac{dW_{fi}}{d(k \cdot k')d\xi} = \frac{\alpha[\int_r^\infty K_{5/3}(y)dy + \kappa r K_{2/3}(r)]}{\sqrt{3}\pi\lambda\!\!\!^{-}_C (k \cdot p_i)^2}, \qquad (3)$$

$$\kappa = \frac{(k \cdot k')\chi_e}{(k \cdot p_i)}, \qquad r = \frac{(k \cdot k')}{\chi_e (k \cdot p_f)},$$

$$\chi_e = \frac{3}{2}(k \cdot p_i)\left|\frac{d\mathbf{a}}{d\xi}\right|\lambda\!\!\!^{-}_C.$$

Here $K_\nu(r)$ is the MacDonald function and $\alpha = e^2/(c\hbar)$.

Collision integral.—In strong fields we introduce χ parameter not only for electrons but also for γ photons and relate the emission probability to $d\chi_\gamma \propto d(k \cdot k')$:

$$\chi_\gamma = \frac{3}{2}(k \cdot k')\left|\frac{d\mathbf{a}}{d\xi}\right|\lambda\!\!\!^{-}_C, \quad \frac{dW_{fi}}{d\chi_\gamma d\xi} = \alpha\left|\frac{d\mathbf{a}}{d\xi}\right| w^{e\rightarrow\gamma,e}_{\chi_e\rightarrow\chi_\gamma}, \qquad (4)$$

$$w^{e\rightarrow\gamma,e}_{\chi_e\rightarrow\chi_\gamma} = \frac{\sqrt{3}}{2\pi\chi_e^2}\left[\chi_\gamma r K_{2/3}(r) + \int_r^\infty K_{5/3}(y)dy\right]. \qquad (5)$$

Here $r = \chi_\gamma/[\chi_e(\chi_e - \chi_\gamma)]$, $\chi_\gamma \leq \chi_e$. The electron parameter, χ_e, is taken for the initial state and its value in the final state is $\chi_e - \chi_\gamma$.

The distribution functions for electrons and photons may be also integrated over $\mathbf{p}_\perp$ and $\mathbf{k}'_\perp$ correspondingly. Thus integrated functions are distributed over $(k \cdot p)$, $(k \cdot k')$. We can parameterize these distributions *locally* via $\chi_e \propto (k \cdot p)$, $\chi_\gamma \propto (k \cdot k')$ and introduce the 1D distribution functions, $f_e(\chi_e)$ and $f_\gamma(\chi_\gamma)$.

The collision integral (see Ref. [15]) describes the change in the particle distributions due to emission and accounts for the electrons, leaving the given phase volume, $d\chi_e$, and those arriving into it within the interval, $d\tilde{\xi} = \alpha|d\mathbf{a}/d\xi|d\xi = 2\alpha c\chi_e d\tau_e/(3\lambda\!\!\!^{-}_C)$:

$$\frac{\delta f_e(\chi_e)}{d\tilde{\xi}} = \int_{\chi_e}^\infty f_e(\chi) w^{e\rightarrow\gamma,e}_{\chi\rightarrow\chi-\chi_e} d\chi - f_e(\chi_e) \times \int_0^{\chi_e} w^{e\rightarrow\gamma,e}_{\chi_e\rightarrow\chi} d\chi, \qquad (6)$$

$$\frac{\delta f_\gamma(\chi_\gamma)}{d\tilde{\xi}} = \int_{\chi_\gamma}^\infty f_e(\chi) w^{e\rightarrow\gamma,e}_{\chi\rightarrow\chi_\gamma} d\chi.$$

Radiation force approximation.—One may exclude the emission of softer γ photons with $\chi_\gamma \leq \varepsilon \ll 1$ from the collision integral by changing the spans as follows:

$$\frac{\delta^+ f_e(\chi_e)}{d\tilde{\xi}} = \int_{\chi_e+\varepsilon}^{\infty} f_e(\chi) w^{e\to\gamma,e}_{\chi\to\chi-\chi_e} d\chi - f_e(\chi_e) \times \int_{\varepsilon}^{\chi_e} w^{e\to\gamma,e}_{\chi_e\to\chi} d\chi, \quad (7)$$

$$\frac{\delta^+ f_\gamma(\chi_\gamma)}{d\tilde{\xi}} = \int_{\chi_\gamma}^{\infty} f_e(\chi) w^{e\to\gamma,e}_{\chi\to\chi_\gamma} d\chi, \qquad \chi_\gamma \geq \varepsilon.$$

The excluded process should be treated separately:

$$\frac{\delta^{(rf)} f_e(\chi_e)}{d\tilde{\xi}} = \frac{\partial}{\partial\chi_e}[w^{(rf)}_{\chi_e} f_e(\chi_e)], \quad (8)$$

$$\frac{\delta^{(rf)} \int_0^{\varepsilon} \chi_\gamma f_\gamma(\chi_\gamma) d\chi_\gamma}{d\tilde{\xi}} = \int_0^{\infty} f_e(\chi_e) w^{(rf)}_{\chi_e} d\chi_e, \quad (9)$$

where the expression for the *radiation force*,

$$w^{(rf)}_{\chi_e} = \int_0^{\varepsilon} \chi_\gamma w^{e\to\gamma,e}_{\chi_e\to\chi_\gamma} d\chi_\gamma, \quad (10)$$

is obtained using the standard Fokker-Planck development (see Ref. [15]) of the collision integral at small $\chi_\gamma \leq \varepsilon$:

$$\int_0^{\varepsilon} [f_e(\chi_e + \chi_\gamma) w^{e\to\gamma,e}_{\chi_e+\chi_\gamma\to\chi_\gamma} - f_e(\chi_e) w^{e\to\gamma,e}_{\chi_e\to\chi_\gamma}] d\chi_\gamma \approx \frac{\partial}{\partial\chi_e}\Big[f_e(\chi_e) \int_0^{\varepsilon} \chi_\gamma w^{e\to\gamma,e}_{\chi_e\to\chi_\gamma} d\chi_\gamma\Big].$$

The advective term like that in Eq. (8), once introduced to the kinetic equation, describes the electron transport along the characteristic lines, $d\chi_e + d\tilde{\xi} w^{(rf)}_{\chi_e} = 0$. This effect is equivalent to that from an extra four-force term, $(dp^\mu/d\tau_e)_{\rm rad}$, in the dynamical equation for the electron four-momentum, p^μ, the force being such that:

$$-w^{(rf)}_{\chi_e} = \frac{d\chi_e}{d\tilde{\xi}} = \frac{\partial\chi_e}{\partial p^\mu}\left(\frac{dp^\mu}{d\tau_e}\right)_{\rm rad} \frac{d\tau_e}{d\tilde{\xi}}. \quad (11)$$

The radiation force is directed along $-p^\mu + k^\mu/(k\cdot p)$. The two terms describe the energy-momentum lost for radiation and that absorbed from the 1D wave field in the course of emission, their total being orthogonal to p^μ (see Ref. [11]). The force magnitude may be found from Eq. (11):

$$\left(\frac{dp^\mu}{d\tau_e}\right)_{\rm rad} = -\frac{2\alpha c}{3\lambda\!\!\!^{-}_C} w^{(rf)}_{\chi_e}\left(p^\mu - \frac{k^\mu}{(k\cdot p)}\right).$$

In the first component of this equation the term $\propto \mathcal{E}$ controls the radiation energy loss rate, $I_{\rm QED}$. In dimensional form and related per time interval, $I_{\rm QED} = -2\alpha m_e c^3 w^{(rf)}_{\chi_e}/(3\lambda\!\!\!^{-}_C)$. At $\chi_e \leq \varepsilon \ll 1$, $I_{\rm QED}$ tends to the expression for the radiation energy loss rate found within the framework of classical electrodynamics. When the radiation force approach is generalized for large $\chi_e \gg 1$, the emission spectrum is modified by QED effects and only a part of this spectrum (which is minor at $\chi_e \gg 1$) is embraced by the radiation force approximation.

Photons in QED-strong fields.—The absorption probability for photons in the 1D field is derived in Appendix B in Ref. [14]. An electron-positron pair (e, p) is generated during photon absorption with the conservation law: $(k\cdot k') = (k\cdot p_e) + (k\cdot p_p)$.

The phase volume element for the created electron, again is chosen in the form $d^2\mathbf{p}_\perp d(k\cdot p)$. The absorption probability, $dW_{fi}/[d\xi d(k\cdot p_e)]$, is integrated over the transversal momentum components and related to the element of the phase volume of the electron, $d(k\cdot p_e)$, resulting in the following collision integral:

$$\frac{\delta^- f_{e,p}(\chi_{e,p})}{d\tilde{\xi}} = \int_{\chi_{e,p}}^{\infty} f_\gamma(\chi_\gamma) w^{\gamma\to e,p}_{\chi_\gamma\to\chi_e} d\chi_\gamma, \quad (12)$$

$$\frac{\delta^- f_\gamma(\chi_\gamma)}{d\tilde{\xi}} = -f_\gamma(\chi_\gamma)\int_0^{\chi_\gamma} w^{\gamma\to e,p}_{\chi_\gamma\to\chi_e} d\chi_e. \quad (13)$$

Here $r = \chi_\gamma/[\chi_e(\chi_\gamma - \chi_e)]$, $\chi_e = \chi_\gamma - \chi_p \leq \chi_\gamma$ and

$$w^{\gamma\to e,p}_{\chi_\gamma\to\chi_e} = \frac{\sqrt{3}}{2\pi\chi_\gamma^2}\Big[\chi_\gamma r K_{2/3}(r) - \int_r^{\infty} K_{5/3}(y) dy\Big]. \quad (14)$$

Solution for kinetic equations.—As long as the distribution functions are integrated over the transversal components of momentum and expressed in terms of the motional integrals, $(k\cdot p_{e,p})$, their evolution is controlled by the collision integrals:

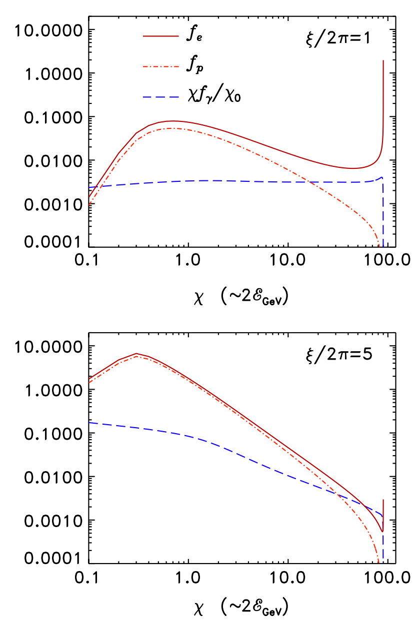

FIG. 1 (color online). Distribution functions of electrons and positrons, $f_{e,p}(\chi)$, and a spectrum of emission, $\chi_\gamma f_\gamma(\chi)/\chi_0$, after the interaction of 46.6 GeV electrons with one cycle (top) and five cycles (bottom) of a laser pulse of intensity $J \approx 5\times 10^{22}$ W/cm^2 [so that $\chi \approx 2\mathcal{E}$[GeV]—see Eq. (2)]. Here $f_e - f_p$ is the distribution of the beam electrons and $\int(f_e - f_p)d\chi = 1$.

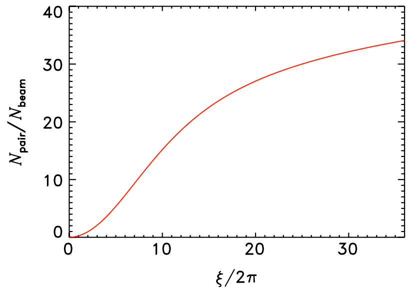

FIG. 2 (color online). Pair production for longer pulse durations, measured in cycles. Other parameters are the same as in Fig. 1.

$$\frac{\partial f_{e,p,\gamma}[\tilde{\xi}, (k \cdot p_{e,p,\gamma})]}{\partial \tilde{\xi}} = (\delta^{(rf)} + \delta^{+} + \delta^{-}) f_{e,p,\gamma}. \quad (15)$$

The derivatives, $\partial/\partial\tilde{\xi}$, are taken at constant $(k \cdot p)$. Equation (15) is easy to solve for the 1D wave field of any shape; however, for a circularly polarized wave of constant amplitude the solution is especially simple. In this case $(k \cdot p)$ differs from χ by a constant factor, and Eq. (15) may be solved with derivatives, $\partial/\partial\tilde{\xi}$, at constant χ for the functions, $f_{e,p,\gamma}(\tilde{\xi}, \chi_{e,p,\gamma})$.

We solve Eq. (15) numerically, by discretizing it on a uniform grid, $\chi_j = j\Delta\chi$, $j = 1, 2, 3, \ldots, N$, with the choice of $\Delta\chi = 0.1$, $\varepsilon = \Delta\chi/2$. The $\tilde{\xi}$-dependent distribution functions on this grid obey the system of $3N$ equations, which is integrated numerically. At initialization, electrons with $f_e(\chi_e) = \delta(\chi_e - \chi_0)$, $\chi_0 = 90$, counter-propagate in the circularly polarized wave field with $|d\mathbf{a}/d\xi| = 110$. This choice corresponds to the electron beam as in Ref. [1] and a laser intensity of $J \approx 5 \times 10^{22}$ W/cm^2 for $\lambda = 0.8$ μm, to be achieved soon.

In Fig. 1 the beam-wave interaction is traced during $\frac{\xi}{2\pi} = 5$ cycles of the incident laser pulse (≈ 13 fs). The initial beam electron energy is rapidly converted into γ photons with high χ_γ, which then rapidly produce pairs, the typical rates of the processes being of the order of the inverse light period. However, the larger fraction of the new particles is born at $\chi \lesssim 1$, with strongly reduced pair production rate. Slow absorption of photons with $\chi_\gamma \sim 1$–2 maintains pair production even after tens of wave periods, as shown in Fig. 2.

Conclusion.—We see that the laser-beam interaction may be accompanied by multiple pair production. The initial energy of a beam electron is efficiently spent for creating pairs with significantly lower energies as well as softer γ photons. This effect may be used for producing a pair plasma. It could also be employed to deactivate after-use electron beams, reducing radiation hazard.

The way to solve the kinetic equations is accurate and does not employ the Monte Carlo method. The solution can be used to benchmark numerical methods designed to simulate processes in QED-strong laser fields.

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