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Validated Solution of Initial Value Problem
for Ordinary Differential Equations
based on Explicit and Implicit Runge-Kutta Schemes¹

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Abstract

We present in this report our tool based on Ibex library which provides an innovative and generic procedure to simulate an ordinary differential equation with any Runge-Kutta scheme (explicit or implicit). Our validated approach is based on the classical two steps integration: the Picard-Lindelöf operator to enclose all the solutions on a one step, and the computation of the approximated solution and its Local Truncating Error. This latter is computed with a generic and elegant approach using interval arithmetic and Fréchet derivatives. We perform a strong experimentation through many numerical experiments coming from three different benchmarks and the results are shown and compared with competition.

Chapter 1

Introduction

Many scientific applications in physical fields such as mechanics, robotics, chemistry or electronics require differential equations. This kind of equations appears when only the velocity and/or the acceleration are available in the modeling of a system. In the general case, these differential equations cannot be formally integrated, i.e., closed form solution are not available, and a numerical integration scheme is used to approximate the state of the system. In this report, we focus on ordinary differential equations for which we develop a new method to solve them and validate the solution.

Notations \dot{y} denotes the time derivative of the function y , i.e., $\frac{dy}{dt}$. x denotes a real values while \mathbf{x} represents a vector of real values. $[x]$ represents an interval values and $[\mathbf{x}]$ represents a vector of interval values.

1.1 Solving ODE with Numerical Methods

An *ordinary differential equation* (ODE for short) is a relation between a function $y : \mathbb{R} \rightarrow \mathbb{R}^n$ and its derivative $\dot{y} = \frac{dy}{dt}$, written as $\dot{y} = f(t, y)$. An *initial value problem* (IVP for short) is an ODE together with an initial condition and a final time

$$\dot{y} = f(t, y) \quad \text{with} \quad y(0) = y_0, \quad y_0 \in \mathbb{R}^n \quad \text{and} \quad t \in [0, t_{\text{end}}] . \quad (1.1)$$

We do not address here the problem of existence of the solution and we shall always assume that $f : \mathbb{R} \times \mathbb{R}^n \rightarrow \mathbb{R}^n$ is continuous in t and globally Lipschitz in y , so Equation (1.1) admits a unique solution on \mathbb{R} , see [10] for more details. As the exact solution $y(t)$ of Equation (1.1) is usually unknown, numerical methods are used to approximate $y(t)$ on a time grid.

1.2 Classical Runge-Kutta methods

We now recall the principles of numerical integration of ordinary differential equations. Solving the IVP means finding a continuous and differentiable function y_∞ such that $y_\infty(0) = y_0$ and

$$\forall t \in [0, t_{\text{end}}], \quad \dot{y}_\infty(t) = f(t, y_\infty(t)) .$$

Note that, higher order differential equations can be translated into first-order ODEs by introducing additional variables for the derivatives of y . We denote the solution at time t of Equation (1.1) with initial condition y_0 at $t = 0$ by $y(t; y_0)$.

An exact solution of Equation (1.1) is rarely computable so that in practice, approximation algorithms are used. The goal of an approximation algorithm is to compute a sequence of $n + 1$ time instants

$$0 = t_0 < t_1 < \dots < t_n = t_{\text{end}},$$

and a sequence of $n + 1$ values y_0, \dots, y_n such that

$$\forall i \in [0, n], \quad y_i \approx y_\infty(t_i; y_0) .$$

There is a huge set of numerical methods to solve Equation (1.1). In this report, we focus on single-step methods member of the Runge-Kutta family, that is these methods only use y_i and approximations of $\dot{y}(t)$ to compute y_{i+1} .

A Runge-Kutta method, starting from an initial value y_n at time t_n and a finite time horizon h , the *step-size*, produces an approximation y_{n+1} at time t_{n+1} , with $t_{n+1} - t_n = h$, of the solution $y(t_{n+1}; y_n)$. Furthermore, to compute y_{n+1} , a Runge-Kutta method computes s evaluations of f at predetermined time instants. The number s is known as the number of *stages* of a Runge-Kutta method. More precisely, a Runge-Kutta method is defined by

$$y_{n+1} = y_n + h \sum_{i=1}^s b_i k_i, \quad (1.2)$$

with k_i defined by

$$k_i = f \left(t_0 + c_i h, y_0 + h \sum_{j=1}^s a_{ij} k_j \right). \quad (1.3)$$

The coefficient c_i , a_{ij} and b_i , for $i, j = 1, 2, \dots, s$, fully characterize the Runge-Kutta methods and their are usually synthesized in a *Butcher tableau* of the form

$$\begin{array}{c|cccc} c_1 & a_{11} & a_{12} & \dots & a_{1s} \\ c_2 & a_{21} & a_{22} & \dots & a_{2s} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ c_s & a_{s1} & a_{s2} & \dots & a_{ss} \\ \hline & b_1 & b_2 & \dots & b_s \end{array}$$

In function of the form of the matrix A , made of the coefficients a_{ij} , a Runge-Kutta method can be

- *explicit*, e.g., the classical Runge-Kutta method of order 4 given in Figure 1.1(a). In other words, the computation of an intermediate k_i only depends on the previous steps k_j for $j < i$;
- *diagonally implicit*, e.g., a diagonally implicit method of order 4 given in Figure 1.1(b). In this case, the computation of an intermediate step k_i involves the value k_i and so non-linear systems in k_i must be solved;
- *fully implicit*, e.g., the Runge-Kutta method with a Lobatto quadrature formula of order 4 given in Figure 1.1(c). In this last case, the computation of intermediate steps involves the solution of a non-linear system of equations in all the values k_i for $i = 1, 2, \dots, s$.

| | | |
|---|--|---|
| $\begin{array}{c cccc} 0 & 0 & 0 & 0 & 0 \\ \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 \\ \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 \\ \frac{1}{2} & 0 & 0 & 1 & 0 \\ \hline & \frac{1}{6} & \frac{1}{3} & \frac{1}{3} & \frac{1}{6} \end{array}$ <p>(a) RK4</p> | $\begin{array}{c ccccc} \frac{1}{4} & \frac{1}{4} & & & \\ \frac{3}{4} & \frac{1}{2} & \frac{1}{4} & & \\ \frac{11}{20} & \frac{17}{50} & \frac{-1}{25} & \frac{1}{4} & \\ \frac{1}{2} & \frac{371}{1360} & \frac{-137}{2720} & \frac{15}{544} & \frac{1}{4} \\ \frac{1}{2} & \frac{25}{24} & \frac{-49}{48} & \frac{125}{16} & \frac{-85}{12} & \frac{1}{4} \\ \hline & \frac{25}{24} & \frac{-49}{48} & \frac{125}{16} & \frac{-85}{12} & \frac{1}{4} \end{array}$ <p>(b) SDIRK4</p> | $\begin{array}{c ccc} 0 & \frac{1}{6} & -\frac{1}{3} & \frac{1}{6} \\ \frac{1}{2} & \frac{1}{6} & \frac{5}{12} & -\frac{1}{12} \\ \frac{1}{2} & \frac{1}{6} & \frac{2}{3} & \frac{1}{6} \\ \hline & \frac{1}{6} & \frac{2}{3} & \frac{1}{6} \end{array}$ <p>(c) Lobatto3c</p> |
|---|--|---|

Figure 1.1: Different kinds of Runge-Kutta methods

Note that in case of implicit Runge-Kutta methods the non-linear systems of n equations must be solved at each integration step. Usually, a Newton-like method is used for this purpose. Nevertheless, such implicit methods have very good stability properties, see [10, Chap. II] for more details, which make them very useful in case of *stiff* ODE.

1.3 Computing with Sets

To take into account numerical approximation coming from floating-point arithmetic and approximation due to numerical integration scheme, set-based computation is required. In this case, we transform an IVP into an interval initial value problem (IIVP for short) that is

$$\dot{y} = f(t, y) \quad \text{with} \quad y(0) = \mathcal{Y}_0, \quad \mathcal{Y}_0 \subseteq \mathbb{R}^n \quad \text{and} \quad t \in [0, t_{\text{end}}] . \quad (1.4)$$

In Equation (1.4), the initial value is given by a set \mathcal{Y}_0 of values, i.e., we do not know exactly the initial value. In other terms, we want to compute the set of solutions $\mathcal{Y}_\infty(t; \mathcal{Y}_0)$ of IIVP such that

$$\mathcal{Y}_\infty(t; \mathcal{Y}_0) = \{y_\infty(t; y_0) : \forall y_0 \in \mathcal{Y}_0\} .$$

Note that the set \mathcal{Y}_∞ should guarantee to contain the true solution y_∞ . For the past decades IIVP have been solved using tools coming from interval analysis. The guaranteed solution of IIVP using interval arithmetic is mainly based on two kinds of methods:

- i) Interval Taylor series methods [14, 13, 1, 15, 11, 18, 6, 12],
- ii) Interval Runge-Kutta methods [8, 3, 2].

The former is the oldest method used in this context. Indeed, R. Moore [14] already applied this method in the sixties and until now it is the most used method to solve Equation (1.4). The latter is more recent, see in particular [3, 2], but Runge-Kutta methods have many interesting properties as strong stability that we would like to exploit in the context of validated solution of ODEs.

We present new guaranteed numerical integration schemes based on implicit Runge-Kutta methods. This work is an extension of [3, 2] which only considered explicit Runge-Kutta methods.

1.3.1 Interval arithmetic

The simplest and most common way to represent and manipulate sets of values is *interval arithmetic* [14]. An interval $[x_i] = [\underline{x}_i, \overline{x}_i]$ defines the set of reals x_i such that $\underline{x}_i \leq x_i \leq \overline{x}_i$. \mathbb{IR} denotes the set of all intervals. The size or the width of $[x_i]$ is denoted by $w([x_i]) = \overline{x}_i - \underline{x}_i$. The center of an interval is denoted by $\text{Mid}([x])$ denotes the middle of $[x]$. A vector of intervals, or a *box*, $[\mathbf{x}]$ is the Cartesian product of intervals $[x_1] \times \dots \times [x_i] \times \dots \times [x_n]$. The width of a box is defined by $w([\mathbf{x}]) = \max_i w([x_i])$.

Interval arithmetic [14] extends to \mathbb{IR} elementary functions over \mathbb{R} . For instance, the interval sum (i.e., $[x_1] + [x_2] = [\underline{x}_1 + \underline{x}_2, \overline{x}_1 + \overline{x}_2]$) encloses the image of the sum function over its arguments, and this enclosing property basically defines what is called an *interval extension* or an *inclusion function*.

Definition 1 (Extension of a function to \mathbb{IR}). *Consider a function $f : \mathbb{R}^n \rightarrow \mathbb{R}$, then $[f] : \mathbb{IR}^n \rightarrow \mathbb{IR}$ is said to be an **extension** of f to intervals if*

$$\begin{aligned} \forall [x] \in \mathbb{IR}^n, \quad [f]([x]) &\supseteq \{f(x), x \in [x]\}, \\ \forall x \in \mathbb{R}^n, \quad f(x) &= [f](x) . \end{aligned}$$

In our context, the expression of a function f is always a composition of elementary functions. The **natural extension** $[f]_N$ is then simply a composition of the corresponding interval operators.

Definition 2 (Overestimation of a set). *Consider the set $\mathcal{F} = \{f(x), x \in [x]\}$, the interval extension $[f]([x])$ is an overestimation of \mathcal{F} and we note*

$$[f]([x]) = \square \mathcal{F} .$$

Definition 3 (Integration). *Let $f : \mathbb{R}^n \rightarrow \mathbb{R}^n$ be a continuous function and $[a] \subset \mathbb{IR}^n$, then the components of $\int_{\underline{a}}^{\overline{a}} f(s) ds$ are*

$$\left\{ \int_{\underline{a}}^{\overline{a}} f(s) ds \right\}_i = \int_{\underline{a}}^{\overline{a}} \{f(s)\}_i ds .$$

where $\{ \}_i$ denotes the i -th component of a vector. Obviously, see [14],

$$\int_{\underline{a}}^{\overline{a}} f(s) ds \in (\underline{a} - \overline{a}) f([a]) = w([a]) [f]([a]) .$$

The *interval arithmetic* is a powerful tool to deal with sets. Nevertheless, this representation usually produces too much over-approximated results, because it cannot take dependencies between variables in account: for instance, if $x = [0, 1]$, then $x - x = [-1, 1] \neq 0$. More generally, it can be shown for most integration schemes that the width of the result can only grow if we interpret sets of values as intervals.

Example 1.3.1. Consider the ordinary differential equation $\dot{x}(t) = -x$ solved with the Euler's method with an initial value ranging in the interval $[0, 1]$ and with a step-size of $h = 0.5$. For one step of integration, we have to compute with interval arithmetic the expression $e = x + h \times (-x)$ which produces as a result the interval $[-0.5, 1]$. Rewriting the expression e such that $e' = x(1-h)$, we obtain the interval $[0, 0.5]$ which is the exact result. Unfortunately, we cannot in general rewrite expressions with only one occurrence of each variable. More generally, it can be shown that for most integration schemes the width of the result can only grow if we interpret sets of values as intervals [16]. ■

1.3.2 Affine arithmetic

To avoid or limit the problem of dependency, we use an improvement over interval arithmetic named *affine arithmetic* [7] which can track linear correlations between variables.

A set of values in this domain is represented by an *affine form* \hat{x} , which is a formal expression of the form

$$\hat{x} = \alpha_0 + \sum_{i=1}^n \alpha_i \varepsilon_i,$$

where the coefficients α_i are real numbers, α_0 being called the *center* of the affine form, and the ε_i are formal variables ranging over the interval $[-1, 1]$ called *noise symbols*.

Obviously, an interval $a = [a_1, a_2]$ can be seen as the affine form $\hat{x} = \alpha_0 + \alpha_1 \varepsilon$ with $\alpha_0 = (a_1 + a_2)/2$ and $\alpha_1 = (a_2 - a_1)/2$. Moreover, affine forms encode linear dependencies between variables: if $x \in [a_1, a_2]$ and y is such that $y = 2x$, then x will be represented by the affine form \hat{x} above and y will be represented as $\hat{y} = 2\alpha_0 + 2\alpha_1 \varepsilon$.

Usual operations on real numbers extend to affine arithmetic in the expected way. For instance, if we have two affine forms $\hat{x} = \alpha_0 + \sum_{i=1}^n \alpha_i \varepsilon_i$ and $\hat{y} = \beta_0 + \sum_{i=1}^n \beta_i \varepsilon_i$, then with $a, b, c \in \mathbb{R}$, we have

$$a\hat{x} \pm b\hat{y} \pm c = (a\alpha_0 \pm b\beta_0 \pm c) + \sum_{i=1}^n (a\alpha_i \pm b\beta_i) \varepsilon_i .$$

However, unlike the affine operations, most operations create new noise symbols. Multiplication for example is defined by

$$\hat{x} \times \hat{y} = \alpha_0 \beta_0 + \sum_{i=1}^n (\alpha_i \beta_0 + \alpha_0 \beta_i) \varepsilon_i + \nu \varepsilon_{n+1},$$

where

$$\nu = \left(\sum_{i=1}^n |\alpha_i| \right) \times \left(\sum_{i=1}^n |\beta_i| \right),$$

over-approximates the error between the linear approximation of multiplication and multiplication itself.

Other operations, as \sin or \exp , are evaluated using two kinds of algorithm: *min range* method and *Chebyshev* method, see [7] for more details. Note that more recent work exists on increasing the accuracy of affine arithmetic [9, 17] but it is not mandatory to consider them in this work.

Example 1.3.2. Consider again $e = x + h \times (-x)$ with $h = 0.5$ and $x = [0, 1]$ which is associated to the affine form $\hat{x} = 0.5 + 0.5\varepsilon_1$. Evaluating e with affine arithmetic without rewriting the expression, we obtain $[0, 0.5]$ as a result. ■

The set-based evaluation of an expression only consists in interpreting all the mathematical operators (such as $+$ or \sin) by their counterpart in affine arithmetic. We will denote by $\text{Aff}(e)$ the evaluation of the expression e using affine arithmetic, see [4] for practical implementation details.

1.4 Scope of the report

In next chapter, we will describe the tool. After a short overview on the verified simulation process (Section 2.1), we will explain our new way to compute the truncating error in Section 2.2. Then, the algorithm used to compute the implicit Runge-Kutta schemes is described (Section 2.3). The chapter 3 gathers a large experimentation in order to compare us to the competition and validated our approach.

Chapter 2

Description of the tool

We describe in this chapter the main contribution of this article that is a new validated method to compute solution of Equation (1.1). Before presenting this new result we recall some results of the validated numerical integration based on Taylor series.

2.1 Overview on verified simulation process

In the classical approach [13, 15] to define validated method for IVP, each step of an integration scheme consists in two steps: *a priori enclosure* and *solution tightening*. Starting from a valid enclosure $[y]_j$ at time t_j , the two following steps are applied

Step 1. Compute an *a priori* enclosure $[\tilde{y}]_j$ of the solution using Banach's theorem and the Picard-Lindelöf operator. This enclosure has the three major properties:

- $y(t, [y]_j)$ is guaranteed to exist for all $t \in [t_j, t_{j+1}]$, i.e., along the current step, and for all $y_j \in [y]_j$.
- $y(t, [y]_j) \subseteq [\tilde{y}]_j$ for all $t \in [t_j, t_{j+1}]$.
- the step-size $h_j = t_{j+1} - t_j$ is as larger as possible in terms of accuracy and existence proof for the IVP solution.

Step 2. Compute a tighter enclosure of $[y]_{j+1}$ such that $y(t_{j+1}, [y]_j) \subseteq [y]_{j+1}$. The main issue in this phase is how to counteract the well known wrapping effect [14, 13, 15]. This phenomenon appears when we try to enclose a set with an interval vector (geometrically a box). The arising overestimation creates a false dynamic for the next step, and, with accumulation, can lead to intervals with an unacceptably large width.

The different enclosures computed during each step are shown on Figure 2.1.

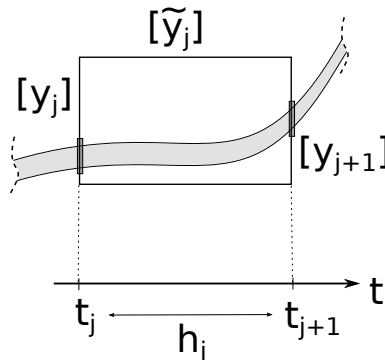


Figure 2.1: Enclosures appeared during one step

Some algorithms useful to perform these two steps are described in the following.

2.1.1 A priori solution enclosure

In order to compute the *a priori* enclosure, we use the Picard-Lindelöf operator. This operator is based on the following theorem.

Theorem 2.1.1 (Banach fixed-point theorem). *Let (K, d) a complete metric space and let $g : K \rightarrow K$ a contraction that is for all x, y in K there exists $c \in]0, 1[$ such that*

$$d(g(x), g(y)) \leq c \cdot d(x, y) ,$$

then g has a unique fixed-point in K .

In context of IVP, we consider the space of continuously differentiable functions $C^0([t_j, t_{j+1}], \mathbb{R}^n)$ and the Picard-Lindelöf operator

$$P_f(y) = t \mapsto y_j + \int_{t_n}^t f(s, y(s)) ds . \quad (2.1)$$

Note that this operator is associated to the integral form of Equation (1.1). So the solution of this operator is also the solution of Equation (1.1).

The Picard-Lindelöf operator is used to check the contraction of the solution on a integration step in order to prove the existence and the uniqueness of the solution of Equation (1.1) as stated by the Banach's fixed-point theorem. Furthermore, this operator is used to compute an enclosure of the solution of IVP over a time interval $[t_j, t_{j+1}]$.

Rectangular method for a priori enclosure

Using interval analysis and with a first order integration scheme we can define a simple interval Picard-Lindelöf operator such that

$$P_f([R]) = [y]_j + [0, h] \cdot f([R]), \quad (2.2)$$

with $h = t_{j+1} - t_j$ the step-size. Theorem 2.1.1 says that if we can find $[R]$ such that $P_f([R]) \subseteq [R]$ then the operator is contracting and Equation (1.1) has a unique solution. Furthermore,

$$\forall t \in [t_j, t_{j+1}], \quad \{y(t; y_j) : \forall y_j \in [y]_j\} \subseteq [R],$$

then $[R]$ is the *a priori* enclosure of the solution of Equation (1.1).

Remark that the operator defined in Equation (2.2) can also define a contractor (in a sens of interval analysis [5]) on $[R]$ after the fixed-point reached such that

$$[R] \leftarrow [R] \cap [y]_j + [0, h] \cdot f([R]) . \quad (2.3)$$

Hence, we can reduce the width of the *a priori* enclosure in order to increase the accuracy of the integration.

The operator defined in Equation (2.2) and its associated contractor defined in Equation (2.3) can be defined over a more accurate integration scheme (at the condition that it is a guaranteed scheme like the interval rectangle rule). For example, the evaluation of $\int_{t_j}^t f(s) ds$ can be easily improved with a Taylor or a Runge-Kutta scheme.

A priori enclosure with Taylor series

Interval version of Taylor series for ODE integration gives

$$[y]_{j+1} \subset \sum_{k=0}^N f^{[k]}([y]_j) h^k + f^{[N+1]}([\tilde{y}]_j) h^{N+1}, \quad (2.4)$$

with $f^{[0]} = [y]_j$, $f^{[1]} = f([y]_j)$, \dots , $f^{[k]} = \frac{1}{k} \left(\frac{\partial f^{[k-1]}}{\partial y} f \right) ([y]_j)$.

By replacing h with interval $[0, h]$, this scheme becomes an efficient Taylor Picard-Lindelöf operator, with a parametric order N such that

$$y_{j+1}([t_j, t_{j+1}]; [R]) = y_j + \sum_{k=0}^N f^{[k]}([y]_j)[0, h^k] + f^{[N+1]}([R])[0, h^{N+1}] . \quad (2.5)$$

In consequence, if $[R] \supseteq y_{j+1}([t_j, t_{j+1}], [R])$, $[R]$ then Equation (2.5) defined a contraction map and Theorem 2.1.1 can be applied.

In our tool, we use it at order 3 by default, it seems to be a good compromise between efficiency and computation quickness.

Note that the scheme defined in Equation (2.4) is usually evaluated in a centered form for a more accurate result

$$[y]_{j+1} \subset \sum_{k=0}^N f^{[k]}(\hat{y}_j)h^k + f^{[N+1]}([\tilde{y}]_j)h^{N+1} + \left(\sum_{k=0}^N J(f^{[k]}, [y]_j)h^k \right) ([y]_j - \hat{y}_j), \quad (2.6)$$

with $\hat{y}_j \in [y]_j$ $J(f^{[k]}, [y]_j)$ is the Jacobian of $f^{[k]}$ evaluated at $[y]_j$. This scheme can also be combined with a QR-factorization to increase stability and counteract wrapping [15]. These two “tricks”, with a strong computational cost, can be avoided by using the affine arithmetic.

Picard-Lindelöf operator, as defined in Equation (2.5), gives an *a priori* enclosure $[R]$, using Theorem 2.1.1. Picard-Lindelöf operator is proven to be contracting on $[R]$, we can then use this operator to contract the box $[R]$ till a fixpoint is reached

In our tool, the default contractor uses a Taylor expansion as follow

$$[R] \cap x_j + \sum_{k=0}^N f^{[k]}([x]_j)[0, h^k] + f^{[N+1]}([R])[0, h^{N+1}]$$

It is very important to contract as much as possible this box $[R]$ because the Taylor remainder is function of $[R]$ and the step-size is function of the Taylor remainder.

A priori enclosure with Runge-Kutta

A novelty of our approach is that we can define a new *a priori* enclosure based on Runge-Kutta methods. The main challenge is to be able to express a formula for the local truncation error (LTE in short). Assuming, we has such formula, we can define a new enclosure such that scheme such that

$$k_i(t, y_j) = f \left(t_j + c_i(t - t_j), y_j + (t - t_j) \sum_{n=1}^s a_{in} k_n \right),$$

$$y_{j+1}(t, \xi) = y_j + (t - t_j) \sum_{i=1}^s b_i k_i(t, y_j) + \text{LTE}(t, y(\xi)) .$$

An inclusion function with $h = t_{j+1} - t_j$ is then defined with

$$y_{j+1}([t_j, t_{j+1}], [R]) = x_j + [0, h] \sum_{i=1}^s b_i k_i([t_j, t_{j+1}], y_j) + \text{LTE}([t_j, t_{j+1}], [R]) .$$

Proving the contraction of such scheme, that is

$$[R] \supseteq x_{j+1}([t_j, t_{j+1}], [R])$$

can prove the existence and the uniqueness of the solution of Equation (1.1) using Theorem 2.1.1. In the sequel of this chapter we present a computable formula of the LTE for any explicit or implicit Runge-Kutta formula.

2.1.2 Tighter enclosure and truncating error

Suppose that Step 1 has been done for the current step and that we dispose of the enclosure $[\tilde{y}]_j$ such that

$$y(t, t_j, [y]_j) \subseteq [\tilde{y}]_j \quad \forall t \in [t_j, t_{j+1}] .$$

In particular, we have $y(t_{j+1}, t_j, [y]_j) \subseteq [\tilde{y}]_j$. The goal of Step 2 is thus to compute the tighter enclosure $[y]_{j+1}$ such that

$$y(t_{j+1}, t_j, [y]_j) \subseteq [y]_{j+1} \subseteq [\tilde{y}]_j .$$

One way to do that consists in computing an approximate solution $y_{j+1} \approx y(t_{j+1}, t_j, [y]_j)$ with an integration scheme $\Phi(t_{j+1}, t_j, [y]_j)$, and then the associated local truncating error $LTE_\Phi(t, t_j, [y]_j)$. Indeed, a guaranteed integration scheme has the property that there exists a time $\xi \in [t_j, t_{j+1}]$ such that

$$y(t_{j+1}, t_j, [y]_j) \subseteq \Phi(t_{j+1}, t_j, [y]_j) + LTE_\Phi(\xi, t_j, [y]_j) \subseteq [\tilde{y}]_j .$$

So $[y]_{j+1} = \Phi(t_{j+1}, t_j, [y]_j) + LTE_\Phi(\xi, t_j, [y]_j)$ is an acceptable tight enclosure.

2.1.3 Wrapping effect

The problem of reducing the wrapping effect has been studied in many different ways. One of the most known and effective is the *QR*-factorization [13]. This method improves the stability of the Taylor series in the Vnode-LP tool [15]. An other way is to modify the geometry of the enclosing set (parallelepipeds with Eijgenram and moore, ellipsoids with Neumaier, convex polygons with Rihm and zonotopes with Stewart and chapoutot).

An efficient affine arithmetic allows us to counteract the wrapping effect as shown in Figure 2.1.3 while keeping a fast computation.

Example 2.1.1. Consider the following IVP

$$\dot{y} = \begin{pmatrix} y_2 \\ -y_1 \end{pmatrix} \quad (2.7)$$

with initial values: $[y_0] = ([-1, 1], [10, 11])$. The exact solution of Equation (2.7) is

$$y(t) = A(t)y_0 \text{ with } A(t) = \begin{pmatrix} \cos(t) & \sin(t) \\ -\sin(t) & \cos(t) \end{pmatrix}$$

We compute periodically at $t = \frac{\pi}{4}n$ with $n = 1, \dots, 4$ the solution of Equation (2.7).

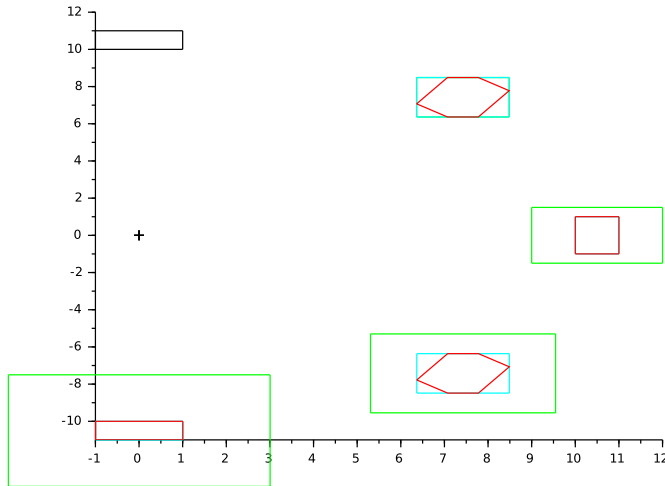


Figure 2.2: Wrapping effect comparison (black: initial, green: interval, blue: interval from QR, red: zonotope from affine)

2.2 Truncating error with elementary derivatives and rooted trees

2.2.1 The local truncating error

The local truncating error, or LTE, is the error due to the integration scheme on one step i , i.e.,

$$\| y(t_j; y_{j-1}) - y_j \| .$$

This error can be bound on each step of integration [10]. The truncating error of a Runge-Kutta scheme $\phi(t) = x_n + (t - t_n) \sum_{i=1}^s b_i k_i(t)$ is obtained by the order condition, and defined by

$$y(t_n; y_{j-1}) - x_j = \frac{h_n^{p+1}}{(p+1)!} \left(f^{(p)}(\xi, y(\xi)) - \frac{d^{p+1}\phi}{dt^{p+1}}(\eta) \right) .$$

This error is exact for one $\xi \in]t_k, t_{k+1}[$ and one $\eta \in]t_n, t_{n+1}[$.

The main issues are then to bound the terms $\frac{d^{p+1}\phi}{dt^{p+1}}(\eta)$ and $f^{(p)}(\xi, x(\xi))$, without knowing ξ and η . Nevertheless, the Picard-Lindelöf operator provides to us the box $y(t, t_j, [y_j]) \subseteq [\tilde{y}_j]$ for all $t \in [t_j, t_{j+1}]$, and so $x(\xi) \in [\tilde{y}_j]$. Obviously, $\eta \in]t_n, t_{n+1}[$, which is well-known.

This approach has given good results, see [2], with $\frac{d^{p+1}\phi}{dt^{p+1}}(\eta)$ computed symbolically. Unfortunately, this computation may take a long time. Moreover, in case of implicit Runge-Kutta method, it is not easy to express ϕ so this approach cannot be applied in that case. We propose an other approach for the computation of the derivatives, based on rooted trees to solve these problems.

2.2.2 Elementary or Frechet's derivatives

To build new Runge-Kutta methods, John Butcher (1963) expressed the Taylor development of the exact solution and the numerical solution from elementary differentials and Frechet derivatives.

Let $z, f(z) \in \mathbb{R}^m$. The M -th Frechet derivative of f is defined by

$$f^{(M)}(z)(K_1, K_2, \dots, K_M) = \sum_{i=1}^m \sum_{j_1=1}^m \sum_{j_2=1}^m \dots \sum_{j_M=1}^m {}^i f_{j_1 j_2 \dots j_M} {}^{j_1} K_1 {}^{j_2} K_2 \dots {}^{j_M} K_M e_i$$

where $K_k = [{}^1 K_1, {}^2 K_2, \dots, {}^M K_M] \in \mathbb{R}^m$, for $k = 1, \dots, M$

The elementary derivatives are defined by ${}^i f_{j_1 j_2 \dots j_M} = \frac{\partial^M}{\partial {}^1 z \partial {}^2 z \dots \partial {}^M z}$.

Example 2.2.1. Let $m = 2$ with $\dot{x} = x^{(1)} = f(x)$ and $M = 1$ then

$$\begin{aligned} f^{(1)}(z)(K_1) &= \sum_{i=1}^2 \sum_{j_1=1}^2 {}^i f_{j_1} ({}^{j_1} K_1) e_i \\ &= \begin{bmatrix} {}^1 f_1 ({}^1 K_1) + {}^1 f_2 ({}^2 K_2) \\ {}^2 f_1 ({}^1 K_1) + {}^2 f_2 ({}^2 K_2) \end{bmatrix} \end{aligned}$$

with ${}^i f_1 = \frac{\partial^i f}{\partial {}^i z}$ and ${}^i f_2 = \frac{\partial^i f}{\partial {}^2 z}$ with $i = 1, 2$

Replacing z by x and K_1 by $f(x)$ we get

$$f^{(1)}(x)(f(x)) = \begin{bmatrix} {}^1 f_1 ({}^1 f) + {}^1 f_2 ({}^2 f) \\ {}^2 f_1 ({}^1 f) + {}^2 f_2 ({}^2 f) \end{bmatrix} = x^{(2)}$$

Hence the second derivative of x is the first Fréchet derivative of f operating on f .

Elementary differentials: The elementary differentials $F_s : \mathbb{R}^m \rightarrow \mathbb{R}^m$ of f and their order are defined recursively by:

1. f is the only elementary differential of order 1
2. if $F_s, s = 1, 2, \dots, M$ are elementary differentials of order r_s then the Fréchet derivative $f^{(M)}(F_1, F_2, \dots, F_m)$ is an elementary differential of order $1 + \sum_{s=1}^M r_s$

Let see different Fréchet derivatives:

- Order 1: f
- Order 2: $f^{(1)}(f)$
- Order 3: $f^{(2)}(f, f) \quad f^{(1)}(f^{(1)}(f))$
- Order 4: $f^{(3)}(f, f, f) \quad f^{(2)}(f, f^{(1)}(f)) \quad f^{(1)}(f^{(2)}(f, f)) \quad f^{(1)}(f^{(1)}(f^{(1)}(f)))$

In consequence,



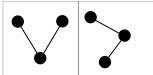
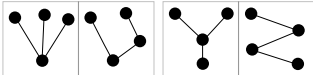
$$x^{(2)} = f^{(1)}(f)$$

















$$x^{(3)} = f^{(2)}(f, f) + f^{(1)}(f^{(1)}(f))$$

2.2.3 Trees

Rooted trees To a given order n , how many elementary differentials do we have?

Note: The answer is the same that counting the number of rooted tree with a given number of nodes (with heap-order).

| Order | Trees | Number of trees |
|-------|---|-----------------|
| 1 |  | 1 |
| 2 |  | 1 |
| 3 |  | 2 |
| 4 |  | 4 |

| Tree | Name | $r(t)$ | $\sigma(t)$ | $\gamma(t)$ | $\alpha(t)$ |
|---|---|-----------------|--------------|-------------|-------------|
|  | τ | 1 | 1 | 1 | 1 |
|  | $[\tau]$ | 2 | 1 | 2 | 1 |
|  | $[\tau^2]$ | 3 | 2 | 3 | 1 |
|  | $[[\tau]]$ | 3 | 1 | 6 | 1 |
|  | $[\tau^3]$ | 4 | 6 | 4 | 1 |
|  | $[\tau[\tau]]$ | 4 | 1 | 8 | 3 |
|  | $[[\tau^2]]$ | 4 | 2 | 12 | 1 |
|  | $[[[\tau]]]$ | 4 | 1 | 24 | 1 |
| Order | Tree | t | $F(t)$ | | |
| 1 |  | τ | f | | |
| 2 |  | $[\tau]$ | $\{f\}$ | | |
| 3 |  | $[\tau^2]$ | $\{f^2\}$ | | |
| 3 |  | $[2\tau]_2$ | $\{2f\}_2$ | | |
| 4 |  | $[\tau^3]$ | $\{f^3\}$ | | |
| 4 |  | $[\tau[\tau]_2$ | $\{f\{f\}_2$ | | |
| 4 |  | $[2\tau^2]_2$ | $\{2f^2\}_2$ | | |
| 4 |  | $[3\tau]_3$ | $\{3f\}_3$ | | |

Elementary differentials and exact solution
Let $\dot{x} = f(x)$, $f : \mathbb{R}^m \rightarrow \mathbb{R}^m$. Then

$$x^{(q)} = \sum_{r(\tau)=q} \alpha(\tau)F(\tau)$$

Note on the number of trees (up to order 11 (left)):

Number of Rooted Trees
1842 719 286 115 48 20 9 4 2 1 1 (total 3047)

Elementary differentials and Runge-Kutta methods

Let the a Runge-Kutta defined by a Butcher table then

$$\frac{d^q}{dh^q} x_n|_{h=0} = \sum_{r(\tau)=q} \alpha(\tau) \gamma(\tau) \psi(\tau) F(\tau)$$

Order Condition The Runge-Kutta method has order p if $\psi(\tau) = \frac{1}{\gamma(\tau)}$ holds for all trees of order $r(\tau) \leq p$ and does not hold for some tree of order $p + 1$.

2.2.4 Local truncating error

Main results: a unified approach

For a Runge-Kutta (explicit and implicit) of order p we have

$$x(t_n; x_0) - x_n = \frac{h^{p+1}}{(p+1)!} \sum_{r(\tau)=p+1} \alpha(\tau) [1 - \gamma(\tau) \psi(\tau)] F(\tau)(x(\xi)) \quad \xi \in [t_n, t_{n+1}]$$

- τ is a **rooted tree**
- $F(\tau)$ is the **elementary differential** associated to τ
- $r(\tau)$ is the order of τ (number of nodes)
- $\gamma(\tau)$ is the density
- $\alpha(\tau)$ is the number of equivalent trees
- $\psi(\tau)$

Note $x(\xi)$ will be over-approximated using Picard-Lindelöf operator.

2.3 Implicit RK computation

2.3.1 Implicit Runge-Kutta methods

Butcher tableau

Implicit Euler The backward Euler method is first order. Unconditionally stable and non-oscillatory for linear diffusion problems.

$$\begin{array}{c|c} 1 & 1 \\ \hline & 1 \end{array}$$

Implicit midpoint The implicit midpoint method is of second order. It is the simplest method in the class of collocation methods known as the Gauss methods. It is a symplectic integrator.

$$\begin{array}{c|c} 1/2 & 1/2 \\ \hline & 1 \end{array}$$

Radau IIA Radau methods are fully implicit methods (matrix A of such methods can have any structure). Radau methods attain order $2s - 1$ for s stages. Radau methods are A-stable, but expensive to implement. Also they can suffer from order reduction. The first order Radau method is similar to backward Euler method.

$$\begin{array}{c|cc} 1/3 & 5/12 & -1/12 \\ 1 & 3/4 & 1/4 \\ \hline & 3/4 & 1/4 \end{array}$$

Lobatto IIIC There are three families of Lobatto methods, called IIIA, IIIB and IIIC. These are named after Rehuel Lobatto. All are implicit methods, have order $2s - 2$ and they all have $c_1 = 0$ and $c_s = 1$. Unlike any explicit method, it's possible for these methods to have the order greater than the number of stages. Lobatto lived before the classic fourth-order method was popularized by Runge and Kutta.

$$\begin{array}{c|ccc} 0 & 1/6 & -1/3 & 1/6 \\ 1/2 & 1/6 & 5/12 & -1/12 \\ 1 & 1/6 & 2/3 & 1/6 \\ \hline & 1/6 & 2/3 & 1/6 \end{array}$$

SDIRK4 For the so-called DIRK methods, also known as SDIRK or semi-explicit or semi-implicit methods, A has a lower triangular structure where the constant in diagonal is chosen for stability reasons. In cases in which the solution of integration in the current step is identical with the final stage, it is possible that a_{11} is equal to 0 rather than to the diagonal value, without taking away from the essential nature of a DIRK method.

$$\begin{array}{c|cccccc} 1/4 & 1/4 & 0 & 0 & 0 & 0 \\ 3/4 & 1/2 & 1/4 & 0 & 0 & 0 \\ 11/20 & 17/50 & -1/25 & 1/4 & 0 & 0 \\ 1/2 & 371/1360 & -137/2720 & 15/544 & 1/4 & 0 \\ 1 & 25/24 & -49/48 & 125/16 & -85/12 & 1/4 \\ \hline & 25/24 & -49/48 & 125/16 & -85/12 & 1/4 \end{array}$$

Properties

2.3.2 Algorithm

Naturally contracting form

$$k_i = f(y_n + h \sum_{j=1}^s a_{i,j} k_j, t_n + c_i h) \quad (2.8)$$

$s = \sum_{j=1}^s a_{i,j} \leq 1$ for all i , then $y_n + h s \subset [\tilde{y}_n]$, then by theorem of ? and Lipschitz condition, f is contracting for k_i .

Algorithm

We use a contractor scheme:

$$k_i \cap f(y_n + h \sum_{j=1}^s a_{i,j} k_j, t_n + c_i h), \quad (2.9)$$

which leads to a fixpoint (one solution by Banach's theorem or Brewer's theorem).

A novelty of our approach is that we can define a new *a priori* enclosure based on Runge-Kutta methods. We can define a new enclosure such that scheme such that

$$\begin{aligned} k_i(t, y_j) &= f \left(t_j + c_i(t - t_j), y_j + (t - t_j) \sum_{n=1}^s a_{in} k_n \right), \\ y_{j+1}(t, \xi) &= y_j + (t - t_j) \sum_{i=1}^s b_i k_i(t, y_j) + \text{LTE}(t, y(\xi)) . \end{aligned}$$

An inclusion function with $h = t_{j+1} - t_j$ is then defined with

$$y_{j+1}([t_j, t_{j+1}], [R]) = x_j + [0, h] \sum_{i=1}^s b_i k_i([t_j, t_{j+1}], y_j) + \text{LTE}([t_j, t_{j+1}], [R]) .$$

Proving the contraction of such scheme, that is

$$[R] \supseteq x_{j+1}([t_j, t_{j+1}], [R])$$

can prove the existence and the uniqueness of the solution of Equation (1.1) using Theorem 2.1.1. In the sequel of this chapter we present a computable formula of the LTE for any explicit or implicit Runge-Kutta formula.

Chapter 3

Experimentation

3.1 Vericomp benchmark

3.1.1 Disclaimer

This section reports the results of the solution of various problems coming from the VERICOMP benchmark¹. For each problem, different validated methods of Runge-Kutta of order 4 are applied among: the classical formula of Runge-Kutta (explicit), the Lobatto-3a formula (implicit) and the Lobatto-3c formula (implicit). Moreover, an homemade version of Taylor series, limited to order 5 and using affine arithmetic, is also applied on each problem.

For each problem, we report the following metrics:

- c5t: user time taken to simulate the problem for 1 second.
- c5w: the final diameter of the solution (infinity norm is used).
- c6t: the time to breakdown the method with a maximal limit of 10 seconds.
- c6w: the diameter of the solution a the breakdown time.

After the results listing, a discussion is done.

3.1.2 Results

¹<http://vericomp.inf.uni-due.de>

Table 3.1: Simulation results of Problem 1

| Problems | Methods | c5t | c5w | c6t | c6w |
|----------|----------------------------|-----------|--------|--------|------------|
| system_1 | TAYLOR4 (TP8) | 0.040 | 5.8147 | 10.000 | 9.6379e+08 |
| system_1 | TAYLOR4 (TP9) | 0.050 | 5.8147 | 10.000 | 9.6379e+08 |
| system_1 | TAYLOR4 (TP10) | 0.060 | 5.8147 | 10.000 | 9.6379e+08 |
| system_1 | TAYLOR4 (TP11) | 0.110 | 5.8147 | 10.000 | 9.6379e+08 |
| system_1 | TAYLOR4 (TP12) | 0.160 | 5.8147 | 10.000 | 9.6379e+08 |
| system_1 | TAYLOR4 (TP13) | 0.220 | 5.8147 | 10.000 | 9.6379e+08 |
| system_1 | TAYLOR4 (TP14) | 0.270 | 5.8147 | 10.000 | 9.6379e+08 |
| system_1 | RK4 (TP8) | 0.030 | 5.8147 | 10.000 | 9.6379e+08 |
| system_1 | RK4 (TP9) | 0.020 | 5.8147 | 10.000 | 9.6379e+08 |
| system_1 | RK4 (TP10) | 0.040 | 5.8147 | 10.000 | 9.6379e+08 |
| system_1 | RK4 (TP11) | 0.080 | 5.8147 | 10.000 | 9.6379e+08 |
| system_1 | RK4 (TP12) | 0.100 | 5.8147 | 10.000 | 9.6379e+08 |
| system_1 | RK4 (TP13) | 0.170 | 5.8147 | 10.000 | 9.6379e+08 |
| system_1 | RK4 (TP14) | 0.230 | 5.8147 | 10.000 | 9.6379e+08 |
| system_1 | LA3 (TP8) | 0.020 | 5.8323 | 10.000 | 9.8667e+08 |
| system_1 | LA3 (TP9) | 0.040 | 5.8253 | 10.000 | 9.774e+08 |
| system_1 | LA3 (TP10) | 0.050 | 5.8212 | 10.000 | 9.7205e+08 |
| system_1 | LA3 (TP11) | 0.070 | 5.8187 | 10.000 | 9.6888e+08 |
| system_1 | LA3 (TP12) | 0.100 | 5.8172 | 10.000 | 9.6695e+08 |
| system_1 | LA3 (TP13) | 0.150 | 5.8163 | 10.000 | 9.6577e+08 |
| system_1 | LA3 (TP14) | 0.200 | 5.8157 | 10.000 | 9.6503e+08 |
| system_1 | LC3 (TP8) | 0.020 | 5.8753 | 10.000 | 1.046e+09 |
| system_1 | LC3 (TP9) | 0.040 | 5.8521 | 10.000 | 1.013e+09 |
| system_1 | LC3 (TP10) | 0.050 | 5.8378 | 10.000 | 9.9387e+08 |
| system_1 | LC3 (TP11) | 0.080 | 5.8291 | 10.000 | 9.8239e+08 |
| system_1 | LC3 (TP12) | 0.120 | 5.8237 | 10.000 | 9.7538e+08 |
| system_1 | LC3 (TP13) | 0.160 | 5.8204 | 10.000 | 9.7105e+08 |
| system_1 | LC3 (TP14) | 0.220 | 5.8183 | 10.000 | 9.6835e+08 |
| system_1 | Riot (02, 1e-11) | 0m1.973s | 10.059 | 10.000 | 1.2112e+10 |
| system_1 | Riot (03, 1e-11) | 0m2.043s | 10.059 | 10.000 | 1.2111e+10 |
| system_1 | Riot (04, 1e-11) | 0m2.102s | 10.059 | 10.000 | 1.2111e+10 |
| system_1 | Riot (05, 1e-11) | 0m2.120s | 10.059 | 10.000 | 1.2111e+10 |
| system_1 | Riot (06, 1e-11) | 0m2.186s | 10.059 | 10.000 | 1.2111e+10 |
| system_1 | Riot (07, 1e-11) | 0m2.270s | 10.059 | 10.000 | 1.2111e+10 |
| system_1 | Riot (09, 1e-11) | 0m23.421s | 10.059 | -0.000 | 1.2111e+10 |
| system_1 | Riot (10, 1e-11) | 0m2.524s | 10.059 | 10.000 | 1.2111e+10 |
| system_1 | Riot (11, 1e-11) | 0m24.797s | 10.059 | -0.000 | 1.2111e+10 |
| system_1 | Riot (15, 1e-11) | 0m2.874s | 10.059 | 10.000 | 1.2111e+10 |
| system_1 | Riot (18, 1e-11) | 0m30.750s | 10.059 | -0.000 | 1.2111e+10 |
| system_1 | Valencia-IVP (0.00025) | 0m1.690s | 4.6755 | 3.469 | 999.98 |
| system_1 | Valencia-IVP (0.0025) | 0m0.157s | 4.7177 | 3.460 | 999.19 |
| system_1 | Valencia-IVP (0.025) | 0m0.022s | 5.1586 | 3.375 | 995.68 |
| system_1 | Valencia-IVP (0.25) | 0m0.010s | 14.082 | 2.250 | 516.32 |
| system_1 | VNODE-LP (12, 1e-1) | 0m0.005s | 6.2022 | 10.000 | 1.6902e+09 |
| system_1 | VNODE-LP (13, 1e-1) | 0m0.008s | 6.9272 | 10.000 | 1.7303e+09 |
| system_1 | VNODE-LP (14, 1e-1) | 0m0.005s | 5.4997 | 10.000 | 1.0761e+09 |
| system_1 | VNODE-LP (15, 1e-14,1e-14) | 0m0.006s | 6.6718 | 10.000 | 1.2705e+09 |
| system_1 | VNODE-LP (20, 1e-14,1e-14) | 0m0.002s | 6.8406 | 10.000 | 1.9442e+09 |
| system_1 | VNODE-LP (25, 1e-14,1e-14) | 0m0.006s | 4.6708 | 10.000 | 4.8518e+08 |

Table 3.2: Simulation results of Problem 2

| Problems | Methods | c5t | c5w | c6t | c6w |
|----------|----------------------------|------------|---------|--------|------------|
| system_2 | TAYLOR4 (TP8) | 0.840 | 0.23254 | 10.000 | 0.00040944 |
| system_2 | TAYLOR4 (TP9) | 1.160 | 0.23254 | 10.000 | 0.00040873 |
| system_2 | TAYLOR4 (TP10) | 1.660 | 0.23254 | 10.000 | 0.00040865 |
| system_2 | TAYLOR4 (TP11) | 2.530 | 0.23254 | 10.000 | 0.00040861 |
| system_2 | TAYLOR4 (TP12) | 3.930 | 0.23254 | 10.000 | 0.0004086 |
| system_2 | TAYLOR4 (TP13) | 6.170 | 0.23254 | 10.000 | 0.0004086 |
| system_2 | TAYLOR4 (TP14) | 9.770 | 0.23254 | 10.000 | 0.0004086 |
| system_2 | RK4 (TP8) | 0.640 | 0.23255 | 10.000 | 0.00040939 |
| system_2 | RK4 (TP9) | 0.890 | 0.23254 | 10.000 | 0.00040875 |
| system_2 | RK4 (TP10) | 1.360 | 0.23254 | 10.000 | 0.00040866 |
| system_2 | RK4 (TP11) | 2.100 | 0.23254 | 10.000 | 0.00040861 |
| system_2 | RK4 (TP12) | 3.240 | 0.23254 | 10.000 | 0.0004086 |
| system_2 | RK4 (TP13) | 5.060 | 0.23254 | 10.000 | 0.0004086 |
| system_2 | RK4 (TP14) | 8.020 | 0.23254 | 10.000 | 0.0004086 |
| system_2 | LA3 (TP8) | 0.500 | 0.26111 | 10.000 | 0.12375 |
| system_2 | LA3 (TP9) | 0.730 | 0.25154 | 10.000 | 0.02491 |
| system_2 | LA3 (TP10) | 1.040 | 0.24447 | 10.000 | 0.010686 |
| system_2 | LA3 (TP11) | 1.600 | 0.24009 | 10.000 | 0.0074653 |
| system_2 | LA3 (TP12) | 2.440 | 0.23734 | 10.000 | 0.0039061 |
| system_2 | LA3 (TP13) | 3.850 | 0.23554 | 10.000 | 0.0074742 |
| system_2 | LA3 (TP14) | 6.100 | 0.23442 | 10.000 | 0.002063 |
| system_2 | LC3 (TP8) | 0.480 | 0.2641 | 10.000 | 0.14326 |
| system_2 | LC3 (TP9) | 0.790 | 0.25281 | 10.000 | 0.014229 |
| system_2 | LC3 (TP10) | 1.130 | 0.24513 | 10.000 | 0.0094465 |
| system_2 | LC3 (TP11) | 1.730 | 0.24048 | 10.000 | 0.011631 |
| system_2 | LC3 (TP12) | 2.700 | 0.23746 | 10.000 | 0.0080097 |
| system_2 | LC3 (TP13) | 4.370 | 0.23561 | 10.000 | 0.0078812 |
| system_2 | LC3 (TP14) | 6.700 | 0.2345 | 10.000 | 0.0017907 |
| system_2 | Riot (03, 1e-11) | 35m43.710s | 0.24697 | 0.000 | 0 |
| system_2 | Riot (05, 1e-11) | 0m0.734s | 0.23588 | 10.000 | 3.4736e+08 |
| system_2 | Riot (06, 1e-11) | 0m0.342s | 0.2417 | -0.000 | 0.2417 |
| system_2 | Riot (07, 1e-11) | 0m9.268s | 0.2417 | -0.000 | 0.42672 |
| system_2 | Riot (10, 1e-11) | 0m0.297s | 0.2417 | 10.000 | 0.43053 |
| system_2 | Riot (15, 1e-11) | 0m0.438s | 0.2417 | 10.000 | 0.69667 |
| system_2 | Valencia-IVP (0.00025) | 0m3.878s | 6.372 | 2.668 | 999.81 |
| system_2 | Valencia-IVP (0.0025) | 0m0.382s | 6.4647 | 2.655 | 992.41 |
| system_2 | Valencia-IVP (0.025) | 0m0.046s | 7.5087 | 2.550 | 986.22 |
| system_2 | VNODE-LP (13, 1e-1) | 0m0.009s | 0.23255 | 10.000 | 0.013215 |
| system_2 | VNODE-LP (15, 1e-14,1e-14) | 0m0.004s | 0.23254 | 10.000 | 0.013205 |
| system_2 | VNODE-LP (20, 1e-14,1e-14) | 0m0.003s | 0.23254 | 10.000 | 0.013205 |
| system_2 | VNODE-LP (25, 1e-14,1e-14) | 0m0.004s | 0.23254 | 10.000 | 0.013205 |

Table 3.3: Simulation results of Problem 3

| Problems | Methods | c5t | c5w | c6t | c6w |
|----------|----------------------------|-----------|---------|--------|----------|
| system_3 | TAYLOR4 (TP8) | 0.060 | 0.48874 | 10.000 | 0.068846 |
| system_3 | TAYLOR4 (TP9) | 0.100 | 0.48163 | 10.000 | 0.065318 |
| system_3 | TAYLOR4 (TP10) | 0.150 | 0.47729 | 10.000 | 0.063275 |
| system_3 | TAYLOR4 (TP11) | 0.200 | 0.47456 | 10.000 | 0.062043 |
| system_3 | TAYLOR4 (TP12) | 0.280 | 0.47286 | 10.000 | 0.06129 |
| system_3 | TAYLOR4 (TP13) | 0.400 | 0.47179 | 10.000 | 0.060825 |
| system_3 | TAYLOR4 (TP14) | 0.000 | 1 | 0.000 | 1 |
| system_3 | RK4 (TP8) | 0.020 | 0.47001 | 10.000 | 0.060058 |
| system_3 | RK4 (TP9) | 0.050 | 0.46999 | 10.000 | 0.060051 |
| system_3 | RK4 (TP10) | 0.090 | 0.46998 | 10.000 | 0.060047 |
| system_3 | RK4 (TP11) | 0.070 | 0.46998 | 10.000 | 0.060046 |
| system_3 | RK4 (TP12) | 0.160 | 0.46998 | 10.000 | 0.060046 |
| system_3 | RK4 (TP13) | 0.220 | 0.46998 | 10.000 | 0.060046 |
| system_3 | RK4 (TP14) | 0.310 | 0.46998 | 10.000 | 0.060045 |
| system_3 | LA3 (TP8) | 0.040 | 0.4851 | 10.000 | 0.068211 |
| system_3 | LA3 (TP9) | 0.050 | 0.47954 | 10.000 | 0.064964 |
| system_3 | LA3 (TP10) | 0.070 | 0.476 | 10.000 | 0.063061 |
| system_3 | LA3 (TP11) | 0.110 | 0.47374 | 10.000 | 0.061905 |
| system_3 | LA3 (TP12) | 0.150 | 0.47235 | 10.000 | 0.061203 |
| system_3 | LA3 (TP13) | 0.200 | 0.47147 | 10.000 | 0.060771 |
| system_3 | LA3 (TP14) | 0.280 | 0.47092 | 10.000 | 0.0605 |
| system_3 | LC3 (TP8) | 0.040 | 0.49094 | 10.000 | 0.071732 |
| system_3 | LC3 (TP9) | 0.060 | 0.4831 | 10.000 | 0.066956 |
| system_3 | LC3 (TP10) | 0.080 | 0.47815 | 10.000 | 0.064212 |
| system_3 | LC3 (TP11) | 0.100 | 0.4751 | 10.000 | 0.062606 |
| system_3 | LC3 (TP12) | 0.150 | 0.47319 | 10.000 | 0.061632 |
| system_3 | LC3 (TP13) | 0.210 | 0.472 | 10.000 | 0.061037 |
| system_3 | LC3 (TP14) | 0.300 | 0.47125 | 10.000 | 0.060666 |
| system_3 | Riot (05, 1e-11) | 0m3.197s | 0.44827 | 10.000 | 0.13094 |
| system_3 | Riot (10, 1e-11) | 0m12.763s | 0.44389 | 10.000 | 0.057421 |
| system_3 | Riot (15, 1e-11) | 0m40.607s | 0.44387 | 10.000 | 0.055362 |
| system_3 | Valencia-IVP (0.00025) | 0m2.780s | 2.8979 | 1.191 | 3.7768 |
| system_3 | Valencia-IVP (0.0025) | 0m0.282s | 2.9052 | 1.175 | 3.694 |
| system_3 | Valencia-IVP (0.025) | 0m0.042s | 2.9872 | 1.300 | 5.8585 |
| system_3 | VNODE-LP (15, 1e-14,1e-14) | 0m0.009s | 0.88761 | 6.361 | 151.77 |
| system_3 | VNODE-LP (20, 1e-14,1e-14) | 0m0.007s | 0.98714 | 3.815 | 218.19 |
| system_3 | VNODE-LP (25, 1e-14,1e-14) | 0m0.009s | 1.1388 | 2.597 | 270.43 |

Table 3.4: Simulation results of Problem 4

| Problems | Methods | c5t | c5w | c6t | c6w |
|----------|----------------------------|-----------|----------|--------|---------|
| system_4 | TAYLOR4 (TP8) | 0.390 | 0.070037 | 9.074 | 85948 |
| system_4 | TAYLOR4 (TP9) | 0.580 | 0.070009 | 9.320 | 62850 |
| system_4 | TAYLOR4 (TP10) | 0.830 | 0.06993 | 8.853 | 85022 |
| system_4 | TAYLOR4 (TP11) | 1.310 | 0.069876 | 7.474 | 67079 |
| system_4 | TAYLOR4 (TP12) | 2.050 | 0.069864 | 8.570 | 70345 |
| system_4 | TAYLOR4 (TP13) | 3.190 | 0.069834 | 8.542 | 64978 |
| system_4 | TAYLOR4 (TP14) | 4.950 | 0.069829 | 7.852 | 73737 |
| system_4 | RK4 (TP8) | 0.240 | 0.069785 | 9.617 | 78366 |
| system_4 | RK4 (TP9) | 0.320 | 0.069787 | 9.191 | 62143 |
| system_4 | RK4 (TP10) | 0.460 | 0.069801 | 8.962 | 77711 |
| system_4 | RK4 (TP11) | 0.670 | 0.069802 | 9.178 | 81171 |
| system_4 | RK4 (TP12) | 1.020 | 0.069819 | 8.626 | 64394 |
| system_4 | RK4 (TP13) | 1.560 | 0.069798 | 8.298 | 82798 |
| system_4 | RK4 (TP14) | 2.370 | 0.06983 | 8.973 | 65817 |
| system_4 | LA3 (TP8) | 0.230 | 0.07624 | 5.512 | 83953 |
| system_4 | LA3 (TP9) | 0.300 | 0.073963 | 5.626 | 82664 |
| system_4 | LA3 (TP10) | 0.390 | 0.072495 | 5.722 | 86373 |
| system_4 | LA3 (TP11) | 0.600 | 0.071545 | 5.928 | 60730 |
| system_4 | LA3 (TP12) | 0.900 | 0.070933 | 5.969 | 81847 |
| system_4 | LA3 (TP13) | 1.360 | 0.07052 | 6.916 | 79535 |
| system_4 | LA3 (TP14) | 2.130 | 0.070275 | 5.983 | 63808 |
| system_4 | LC3 (TP8) | 0.200 | 0.077751 | 5.516 | 97508 |
| system_4 | LC3 (TP9) | 0.280 | 0.074792 | 5.726 | 88836 |
| system_4 | LC3 (TP10) | 0.380 | 0.073062 | 5.658 | 74922 |
| system_4 | LC3 (TP11) | 0.570 | 0.071849 | 5.816 | 95737 |
| system_4 | LC3 (TP12) | 0.790 | 0.071113 | 6.249 | 82501 |
| system_4 | LC3 (TP13) | 1.290 | 0.070648 | 6.607 | 67028 |
| system_4 | LC3 (TP14) | 1.980 | 0.070313 | 7.398 | 68298 |
| system_4 | Riot (05, 1e-11) | 0m37.601s | 0.06757 | 0.000 | 0 |
| system_4 | Riot (10, 1e-11) | 0m3.171s | 0.06757 | 10.000 | 0.18331 |
| system_4 | Riot (15, 1e-11) | 0m9.102s | 0.06757 | 10.000 | 0.30021 |
| system_4 | Valencia-IVP (0.00025) | 0m5.231s | 10.971 | 1.140 | 910.02 |
| system_4 | Valencia-IVP (0.0025) | 0m0.679s | 13.023 | 1.105 | 154.09 |
| system_4 | Valencia-IVP (0.025) | 0m0.063s | 3.2425 | 0.600 | 3.2425 |
| system_4 | VNODE-LP (15, 1e-14,1e-14) | 0m0.012s | 0.073974 | 5.055 | 10185 |
| system_4 | VNODE-LP (20, 1e-14,1e-14) | 0m0.014s | 0.075043 | 4.977 | 21260 |
| system_4 | VNODE-LP (25, 1e-14,1e-14) | 0m0.012s | 0.076265 | 4.913 | 30511 |

Table 3.5: Simulation results of Problem 7

| Problems | Methods | c5t | c5w | c6t | c6w |
|----------|----------------------------|----------|------------|--------|------------|
| system_7 | TAYLOR4 (TP8) | 0.000 | 5.4885e-09 | 10.000 | 5.2398e-09 |
| system_7 | TAYLOR4 (TP9) | 0.000 | 5.6577e-10 | 10.000 | 5.4977e-10 |
| system_7 | TAYLOR4 (TP10) | 0.010 | 5.8386e-11 | 10.000 | 5.3574e-11 |
| system_7 | TAYLOR4 (TP11) | 0.010 | 5.9324e-12 | 10.000 | 5.5432e-12 |
| system_7 | TAYLOR4 (TP12) | 0.020 | 6.4071e-13 | 10.000 | 5.8407e-13 |
| system_7 | TAYLOR4 (TP13) | 0.030 | 1.3856e-13 | 10.000 | 5.8756e-14 |
| system_7 | TAYLOR4 (TP14) | 0.050 | 1.2923e-13 | 10.000 | 5.9005e-15 |
| system_7 | RK4 (TP8) | 0.000 | 6.9766e-09 | 10.000 | 6.05e-09 |
| system_7 | RK4 (TP9) | 0.000 | 7.3286e-10 | 10.000 | 6.93e-10 |
| system_7 | RK4 (TP10) | 0.000 | 7.5791e-11 | 10.000 | 7.3548e-11 |
| system_7 | RK4 (TP11) | 0.010 | 7.7225e-12 | 10.000 | 7.2765e-12 |
| system_7 | RK4 (TP12) | 0.010 | 7.8859e-13 | 10.000 | 7.4488e-13 |
| system_7 | RK4 (TP13) | 0.020 | 1.0791e-13 | 10.000 | 7.5389e-14 |
| system_7 | RK4 (TP14) | 0.030 | 5.6066e-14 | 10.000 | 7.6827e-15 |
| system_7 | LA3 (TP8) | 0.000 | 5.199e-09 | 10.000 | 5.0889e-09 |
| system_7 | LA3 (TP9) | 0.000 | 5.4665e-10 | 10.000 | 4.8474e-10 |
| system_7 | LA3 (TP10) | 0.000 | 5.792e-11 | 10.000 | 5.61e-11 |
| system_7 | LA3 (TP11) | 0.000 | 5.7909e-12 | 10.000 | 5.4252e-12 |
| system_7 | LA3 (TP12) | 0.010 | 6.0674e-13 | 10.000 | 5.8379e-13 |
| system_7 | LA3 (TP13) | 0.020 | 8.2267e-14 | 10.000 | 5.7728e-14 |
| system_7 | LA3 (TP14) | 0.030 | 4.13e-14 | 10.000 | 5.8007e-15 |
| system_7 | LC3 (TP8) | 0.000 | 5.362e-09 | 10.000 | 5.0148e-09 |
| system_7 | LC3 (TP9) | 0.000 | 5.611e-10 | 10.000 | 5.5022e-10 |
| system_7 | LC3 (TP10) | 0.000 | 5.8373e-11 | 10.000 | 5.2443e-11 |
| system_7 | LC3 (TP11) | 0.010 | 5.8898e-12 | 10.000 | 5.6076e-12 |
| system_7 | LC3 (TP12) | 0.010 | 6.0607e-13 | 10.000 | 5.6303e-13 |
| system_7 | LC3 (TP13) | 0.020 | 8.4266e-14 | 10.000 | 5.7818e-14 |
| system_7 | LC3 (TP14) | 0.040 | 4.4076e-14 | 10.000 | 5.8898e-15 |
| system_7 | Riot (05, 1e-11) | 0m0.073s | 1.8582e-11 | 1.000 | 1.8582e-11 |
| system_7 | Riot (10, 1e-11) | 0m0.106s | 1.199e-14 | 10.000 | 1.061e-12 |
| system_7 | Riot (15, 1e-11) | 0m0.189s | 1.7097e-14 | 0.000 | 0 |
| system_7 | Valencia-IVP (0.00025) | 0m1.491s | 0.00029389 | 10.000 | 2.7571 |
| system_7 | Valencia-IVP (0.0025) | 0m0.132s | 0.0029465 | 10.000 | 27.915 |
| system_7 | Valencia-IVP (0.025) | 0m0.016s | 0.030251 | 10.000 | 316.61 |
| system_7 | VNODE-LP (15, 1e-14,1e-14) | 0m0.005s | 1.6653e-16 | 10.000 | 4.6756e-19 |
| system_7 | VNODE-LP (20, 1e-14,1e-14) | 0m0.003s | 2.7756e-16 | 10.000 | 4.0658e-19 |
| system_7 | VNODE-LP (25, 1e-14,1e-14) | 0m0.007s | 1.6653e-16 | 10.000 | 2.9138e-19 |

Table 3.6: Simulation results of Problem 8

| Problems | Methods | c5t | c5w | c6t | c6w |
|----------|----------------------------|----------|------------|--------|------------|
| system_8 | TAYLOR4 (TP8) | 0.630 | 6.2392e-08 | 10.000 | 2.6753e-07 |
| system_8 | TAYLOR4 (TP9) | 0.900 | 6.8627e-09 | 10.000 | 7.328e-08 |
| system_8 | TAYLOR4 (TP10) | 1.340 | 7.1243e-10 | 10.000 | 1.0083e-08 |
| system_8 | TAYLOR4 (TP11) | 2.100 | 7.4399e-11 | 10.000 | 1.343e-09 |
| system_8 | TAYLOR4 (TP12) | 3.380 | 7.6358e-12 | 10.000 | 1.7369e-10 |
| system_8 | TAYLOR4 (TP13) | 5.260 | 1.0223e-12 | 10.000 | 2.2065e-11 |
| system_8 | TAYLOR4 (TP14) | 8.140 | 5.7332e-13 | 10.000 | 3.1279e-12 |
| system_8 | RK4 (TP8) | 0.510 | 8.0492e-08 | 10.000 | 4.8703e-07 |
| system_8 | RK4 (TP9) | 0.760 | 8.8927e-09 | 10.000 | 9.2522e-08 |
| system_8 | RK4 (TP10) | 1.140 | 9.2505e-10 | 10.000 | 1.1545e-08 |
| system_8 | RK4 (TP11) | 1.810 | 9.6979e-11 | 10.000 | 1.3574e-09 |
| system_8 | RK4 (TP12) | 2.810 | 9.8163e-12 | 10.000 | 1.8886e-10 |
| system_8 | RK4 (TP13) | 4.420 | 1.0665e-12 | 10.000 | 2.5177e-11 |
| system_8 | RK4 (TP14) | 6.910 | 2.8466e-13 | 10.000 | 3.3497e-12 |
| system_8 | LA3 (TP8) | 0.410 | 6.3861e-08 | 10.000 | 1.9173e-06 |
| system_8 | LA3 (TP9) | 0.590 | 6.8303e-09 | 10.000 | 2.1645e-07 |
| system_8 | LA3 (TP10) | 0.870 | 7.1757e-10 | 10.000 | 2.0083e-08 |
| system_8 | LA3 (TP11) | 1.320 | 7.3416e-11 | 10.000 | 1.9068e-09 |
| system_8 | LA3 (TP12) | 2.100 | 7.5049e-12 | 10.000 | 2.0342e-10 |
| system_8 | LA3 (TP13) | 3.280 | 8.1635e-13 | 10.000 | 2.2924e-11 |
| system_8 | LA3 (TP14) | 5.150 | 2.1383e-13 | 10.000 | 2.7943e-12 |
| system_8 | LC3 (TP8) | 0.430 | 6.3703e-08 | 10.000 | 3.2935e-06 |
| system_8 | LC3 (TP9) | 0.630 | 6.9067e-09 | 10.000 | 2.6899e-07 |
| system_8 | LC3 (TP10) | 0.950 | 7.17e-10 | 10.000 | 2.3447e-08 |
| system_8 | LC3 (TP11) | 1.460 | 7.3931e-11 | 10.000 | 2.107e-09 |
| system_8 | LC3 (TP12) | 2.300 | 7.5591e-12 | 10.000 | 2.1838e-10 |
| system_8 | LC3 (TP13) | 3.630 | 8.2462e-13 | 10.000 | 2.4242e-11 |
| system_8 | LC3 (TP14) | 5.610 | 2.2604e-13 | 10.000 | 2.9331e-12 |
| system_8 | Riot (05, 1e-11) | 0m0.296s | 9.0226e-11 | 10.000 | 8.8003e-05 |
| system_8 | Riot (10, 1e-11) | 0m0.207s | 1.299e-14 | 10.000 | 1.3371e-10 |
| system_8 | Riot (15, 1e-11) | 0m0.253s | 1.8319e-14 | 10.000 | 8.3085e-15 |
| system_8 | Valencia-IVP (0.00025) | 0m4.114s | 0.0026387 | 5.269 | 999.48 |
| system_8 | Valencia-IVP (0.0025) | 0m0.402s | 0.026723 | 4.485 | 996.18 |
| system_8 | Valencia-IVP (0.025) | 0m0.048s | 0.30489 | 3.575 | 963.25 |
| system_8 | VNODE-LP (15, 1e-14,1e-14) | 0m0.006s | 2.1094e-15 | 10.000 | 2.3327e-16 |
| system_8 | VNODE-LP (20, 1e-14,1e-14) | 0m0.005s | 1.1102e-15 | 10.000 | 1.0988e-16 |
| system_8 | VNODE-LP (25, 1e-14,1e-14) | 0m0.003s | 8.8818e-16 | 10.000 | 8.5489e-17 |

Table 3.7: Simulation results of Problem 10

| Problems | Methods | c5t | c5w | c6t | c6w |
|-----------|----------------------------|----------|------------|--------|------------|
| system_10 | TAYLOR4 (TP8) | 0.010 | 2.1154e-08 | 10.000 | 2.5347e-08 |
| system_10 | TAYLOR4 (TP9) | 0.020 | 2.2594e-09 | 10.000 | 2.6471e-09 |
| system_10 | TAYLOR4 (TP10) | 0.030 | 2.3767e-10 | 10.000 | 2.7776e-10 |
| system_10 | TAYLOR4 (TP11) | 0.050 | 2.4321e-11 | 10.000 | 2.8726e-11 |
| system_10 | TAYLOR4 (TP12) | 0.090 | 2.5682e-12 | 10.000 | 2.9864e-12 |
| system_10 | TAYLOR4 (TP13) | 0.140 | 3.8791e-13 | 10.000 | 4.0901e-13 |
| system_10 | TAYLOR4 (TP14) | 0.230 | 2.4336e-13 | 10.000 | 2.105e-13 |
| system_10 | RK4 (TP8) | 0.000 | 4.9113e-08 | 10.000 | 6.3159e-08 |
| system_10 | RK4 (TP9) | 0.010 | 5.258e-09 | 10.000 | 6.5608e-09 |
| system_10 | RK4 (TP10) | 0.010 | 5.1864e-10 | 10.000 | 6.569e-10 |
| system_10 | RK4 (TP11) | 0.010 | 4.895e-11 | 10.000 | 6.0076e-11 |
| system_10 | RK4 (TP12) | 0.020 | 4.5011e-12 | 10.000 | 5.4561e-12 |
| system_10 | RK4 (TP13) | 0.040 | 4.3721e-13 | 10.000 | 5.1514e-13 |
| system_10 | RK4 (TP14) | 0.060 | 7.1054e-14 | 10.000 | 7.272e-14 |
| system_10 | LA3 (TP8) | 0.000 | 1.9603e-08 | 10.000 | 2.3468e-08 |
| system_10 | LA3 (TP9) | 0.010 | 2.1781e-09 | 10.000 | 2.5435e-09 |
| system_10 | LA3 (TP10) | 0.010 | 2.278e-10 | 10.000 | 2.705e-10 |
| system_10 | LA3 (TP11) | 0.020 | 2.4233e-11 | 10.000 | 2.8082e-11 |
| system_10 | LA3 (TP12) | 0.040 | 2.478e-12 | 10.000 | 2.9076e-12 |
| system_10 | LA3 (TP13) | 0.060 | 2.7711e-13 | 10.000 | 3.1497e-13 |
| system_10 | LA3 (TP14) | 0.090 | 6.8168e-14 | 10.000 | 6.5503e-14 |
| system_10 | LC3 (TP8) | 0.000 | 2.6295e-08 | 10.000 | 3.4923e-08 |
| system_10 | LC3 (TP9) | 0.010 | 3.0011e-09 | 10.000 | 3.521e-09 |
| system_10 | LC3 (TP10) | 0.010 | 2.8753e-10 | 10.000 | 3.508e-10 |
| system_10 | LC3 (TP11) | 0.020 | 2.8342e-11 | 10.000 | 3.4456e-11 |
| system_10 | LC3 (TP12) | 0.030 | 2.7964e-12 | 10.000 | 3.3326e-12 |
| system_10 | LC3 (TP13) | 0.050 | 2.9554e-13 | 10.000 | 3.4062e-13 |
| system_10 | LC3 (TP14) | 0.070 | 6.0396e-14 | 10.000 | 5.9508e-14 |
| system_10 | Riot (05, 1e-11) | 0m0.148s | 3.2904e-11 | 10.000 | 4.4509e-11 |
| system_10 | Riot (10, 1e-11) | 0m0.154s | 2.276e-14 | 10.000 | 2.4266e-12 |
| system_10 | Riot (15, 1e-11) | 0m0.235s | 2.1427e-14 | 10.000 | 2.0872e-14 |
| system_10 | Valencia-IVP (0.00025) | 0m1.280s | 0.00015473 | 10.000 | 0.0022794 |
| system_10 | Valencia-IVP (0.0025) | 0m0.111s | 0.0015521 | 10.000 | 0.022876 |
| system_10 | Valencia-IVP (0.025) | 0m0.014s | 0.016012 | 10.000 | 0.23397 |
| system_10 | VNODE-LP (15, 1e-14,1e-14) | 0m0.007s | 1.6653e-15 | 10.000 | 1.4988e-15 |
| system_10 | VNODE-LP (20, 1e-14,1e-14) | 0m0.006s | 1.2212e-15 | 10.000 | 1.1102e-15 |
| system_10 | VNODE-LP (25, 1e-14,1e-14) | 0m0.004s | 9.992e-16 | 10.000 | 1.1102e-15 |

Table 3.8: Simulation results of Problem 11

| Problems | Methods | c5t | c5w | c6t | c6w |
|-----------|----------------------------|----------|------------|--------|------------|
| system_11 | TAYLOR4 (TP8) | 0.260 | 1.5364e-07 | 10.000 | 0.00011249 |
| system_11 | TAYLOR4 (TP9) | 0.380 | 1.6536e-08 | 10.000 | 0.0001409 |
| system_11 | TAYLOR4 (TP10) | 0.600 | 1.6928e-09 | 10.000 | 6.8266e-05 |
| system_11 | TAYLOR4 (TP11) | 0.950 | 1.7436e-10 | 10.000 | 7.4563e-06 |
| system_11 | TAYLOR4 (TP12) | 1.490 | 1.8469e-11 | 10.000 | 7.9824e-07 |
| system_11 | TAYLOR4 (TP13) | 2.280 | 2.9283e-12 | 10.000 | 1.0116e-07 |
| system_11 | TAYLOR4 (TP14) | 3.610 | 1.9837e-12 | 10.000 | 3.6623e-08 |
| system_11 | RK4 (TP8) | 0.160 | 1.4924e-07 | 10.000 | 9.9104e-05 |
| system_11 | RK4 (TP9) | 0.240 | 1.6173e-08 | 10.000 | 0.00010979 |
| system_11 | RK4 (TP10) | 0.370 | 1.6512e-09 | 10.000 | 3.7122e-05 |
| system_11 | RK4 (TP11) | 0.560 | 1.6831e-10 | 10.000 | 4.4121e-06 |
| system_11 | RK4 (TP12) | 0.910 | 1.7229e-11 | 10.000 | 4.5013e-07 |
| system_11 | RK4 (TP13) | 1.390 | 2.037e-12 | 10.000 | 5.0184e-08 |
| system_11 | RK4 (TP14) | 2.130 | 6.8701e-13 | 10.000 | 1.2723e-08 |
| system_11 | LA3 (TP8) | 0.150 | 1.3016e-07 | 10.000 | 0.00027567 |
| system_11 | LA3 (TP9) | 0.210 | 1.3811e-08 | 10.000 | 5.0329e-05 |
| system_11 | LA3 (TP10) | 0.320 | 1.5537e-09 | 10.000 | 3.3377e-05 |
| system_11 | LA3 (TP11) | 0.500 | 1.6718e-10 | 10.000 | 4.3944e-06 |
| system_11 | LA3 (TP12) | 0.790 | 1.5877e-11 | 10.000 | 4.2412e-07 |
| system_11 | LA3 (TP13) | 1.240 | 1.8541e-12 | 10.000 | 4.6319e-08 |
| system_11 | LA3 (TP14) | 1.890 | 5.9908e-13 | 10.000 | 1.1111e-08 |
| system_11 | LC3 (TP8) | 0.140 | 1.2294e-07 | 10.000 | 0.00022257 |
| system_11 | LC3 (TP9) | 0.200 | 1.2053e-08 | 10.000 | 5.6171e-05 |
| system_11 | LC3 (TP10) | 0.310 | 1.1696e-09 | 10.000 | 4.3e-05 |
| system_11 | LC3 (TP11) | 0.470 | 1.1365e-10 | 10.000 | 4.8341e-06 |
| system_11 | LC3 (TP12) | 0.740 | 1.1288e-11 | 10.000 | 4.7542e-07 |
| system_11 | LC3 (TP13) | 1.160 | 1.3585e-12 | 10.000 | 5.0885e-08 |
| system_11 | LC3 (TP14) | 1.770 | 5.1648e-13 | 10.000 | 1.1353e-08 |
| system_11 | Riot (05, 1e-11) | 0m0.593s | 3.3225e-10 | 10.000 | 3.6967e-08 |
| system_11 | Riot (10, 1e-11) | 0m0.299s | 6.505e-12 | 10.000 | 3.2633e-09 |
| system_11 | Riot (15, 1e-11) | 0m0.436s | 3.5971e-14 | 10.000 | 5.0365e-10 |
| system_11 | Valencia-IVP (0.00025) | 0m1.732s | 0.011564 | 4.825 | 986.14 |
| system_11 | Valencia-IVP (0.0025) | 0m0.252s | 0.11774 | 2.902 | 1.5629 |
| system_11 | Valencia-IVP (0.025) | 0m0.094s | 1.5234 | 1.050 | 1.7124 |
| system_11 | VNODE-LP (15, 1e-14,1e-14) | 0m0.015s | 1.3101e-14 | 10.000 | 2.7778e-12 |
| system_11 | VNODE-LP (20, 1e-14,1e-14) | 0m0.013s | 9.1038e-15 | 10.000 | 1.9398e-12 |
| system_11 | VNODE-LP (25, 1e-14,1e-14) | 0m0.011s | 6.8834e-15 | 10.000 | 2.2919e-12 |

Table 3.9: Simulation results of Problem 13

| Problems | Methods | c5t | c5w | c6t | c6w |
|-----------|----------------------------|----------|------------|--------|------------|
| system_13 | TAYLOR4 (TP8) | 0.100 | 6.0623e-08 | 10.000 | 1.1392e-05 |
| system_13 | TAYLOR4 (TP9) | 0.160 | 6.3074e-09 | 10.000 | 6.6022e-06 |
| system_13 | TAYLOR4 (TP10) | 0.260 | 6.6362e-10 | 10.000 | 6.3809e-06 |
| system_13 | TAYLOR4 (TP11) | 0.410 | 6.9288e-11 | 10.000 | 5.969e-06 |
| system_13 | TAYLOR4 (TP12) | 0.630 | 8.6562e-12 | 10.000 | 5.8669e-06 |
| system_13 | TAYLOR4 (TP13) | 0.990 | 3.336e-12 | 10.000 | 9.4036e-06 |
| system_13 | TAYLOR4 (TP14) | 1.570 | 4.281e-12 | 10.000 | 2.4348e-05 |
| system_13 | RK4 (TP8) | 0.070 | 7.7716e-08 | 10.000 | 1.1601e-05 |
| system_13 | RK4 (TP9) | 0.120 | 8.0154e-09 | 10.000 | 2.9548e-06 |
| system_13 | RK4 (TP10) | 0.180 | 8.5062e-10 | 10.000 | 3.2373e-06 |
| system_13 | RK4 (TP11) | 0.290 | 8.8824e-11 | 10.000 | 4.3262e-06 |
| system_13 | RK4 (TP12) | 0.440 | 9.7406e-12 | 10.000 | 5.0541e-06 |
| system_13 | RK4 (TP13) | 0.690 | 1.9238e-12 | 10.000 | 4.0228e-06 |
| system_13 | RK4 (TP14) | 1.100 | 1.6866e-12 | 10.000 | 1.052e-05 |
| system_13 | LA3 (TP8) | 0.060 | 5.6343e-08 | 10.000 | 2.5172e-05 |
| system_13 | LA3 (TP9) | 0.090 | 6.0874e-09 | 10.000 | 1.0084e-05 |
| system_13 | LA3 (TP10) | 0.140 | 6.5448e-10 | 10.000 | 5.8655e-06 |
| system_13 | LA3 (TP11) | 0.220 | 6.8319e-11 | 10.000 | 5.7753e-06 |
| system_13 | LA3 (TP12) | 0.350 | 7.3896e-12 | 10.000 | 4.6608e-06 |
| system_13 | LA3 (TP13) | 0.530 | 1.4424e-12 | 10.000 | 3.0252e-06 |
| system_13 | LA3 (TP14) | 0.830 | 1.2559e-12 | 10.000 | 3.7585e-06 |
| system_13 | LC3 (TP8) | 0.060 | 5.7775e-08 | 10.000 | 3.7157e-05 |
| system_13 | LC3 (TP9) | 0.100 | 6.2167e-09 | 10.000 | 1.62e-05 |
| system_13 | LC3 (TP10) | 0.150 | 6.5544e-10 | 10.000 | 7.0966e-06 |
| system_13 | LC3 (TP11) | 0.250 | 6.8894e-11 | 10.000 | 7.2423e-06 |
| system_13 | LC3 (TP12) | 0.390 | 7.4376e-12 | 10.000 | 6.7877e-06 |
| system_13 | LC3 (TP13) | 0.590 | 1.5206e-12 | 10.000 | 1.347e-05 |
| system_13 | LC3 (TP14) | 0.920 | 1.3589e-12 | 10.000 | 9.5534e-06 |
| system_13 | Riot (05, 1e-11) | 0m0.182s | 2.3274e-10 | 10.000 | 2.2851e-09 |
| system_13 | Riot (10, 1e-11) | 0m0.119s | 3.5083e-14 | 10.000 | 1.236e-10 |
| system_13 | Riot (15, 1e-11) | 0m0.153s | 1.1813e-13 | 10.000 | 5.4101e-12 |
| system_13 | Valencia-IVP (0.00025) | 0m1.141s | 0.0044966 | 7.088 | 999.86 |
| system_13 | Valencia-IVP (0.0025) | 0m0.099s | 0.045269 | 5.923 | 999.03 |
| system_13 | Valencia-IVP (0.025) | 0m0.017s | 0.48459 | 4.650 | 990.84 |
| system_13 | VNODE-LP (15, 1e-14,1e-14) | 0m0.004s | 6.2172e-15 | 10.000 | 2.4802e-13 |
| system_13 | VNODE-LP (20, 1e-14,1e-14) | 0m0.005s | 3.9968e-15 | 10.000 | 2.3404e-13 |
| system_13 | VNODE-LP (25, 1e-14,1e-14) | 0m0.005s | 1.7764e-15 | 10.000 | 1.1502e-13 |

Table 3.10: Simulation results of Problem 14

| Problems | Methods | c5t | c5w | c6t | c6w |
|-----------|----------------------------|----------|------------|--------|-------------|
| system_14 | TAYLOR4 (TP8) | 0.180 | 6.7792e-07 | 10.000 | 3.6732e+06 |
| system_14 | TAYLOR4 (TP9) | 0.260 | 6.9365e-08 | 10.000 | 3.7168e+05 |
| system_14 | TAYLOR4 (TP10) | 0.420 | 6.9965e-09 | 10.000 | 37470 |
| system_14 | TAYLOR4 (TP11) | 0.640 | 7.1965e-10 | 10.000 | 3831 |
| system_14 | TAYLOR4 (TP12) | 1.000 | 9.1987e-11 | 10.000 | 487.39 |
| system_14 | TAYLOR4 (TP13) | 1.570 | 4.0941e-11 | 10.000 | 212.59 |
| system_14 | TAYLOR4 (TP14) | 2.460 | 5.42e-11 | 10.000 | 280.91 |
| system_14 | RK4 (TP8) | 0.140 | 8.8443e-07 | 10.000 | 4.8078e+06 |
| system_14 | RK4 (TP9) | 0.210 | 9.0238e-08 | 10.000 | 4.8664e+05 |
| system_14 | RK4 (TP10) | 0.330 | 9.1356e-09 | 10.000 | 49032 |
| system_14 | RK4 (TP11) | 0.520 | 9.2979e-10 | 10.000 | 4954.2 |
| system_14 | RK4 (TP12) | 0.830 | 1.0077e-10 | 10.000 | 536.16 |
| system_14 | RK4 (TP13) | 1.250 | 2.2155e-11 | 10.000 | 116.14 |
| system_14 | RK4 (TP14) | 1.980 | 2.1288e-11 | 10.000 | 110.34 |
| system_14 | LA3 (TP8) | 0.110 | 6.5762e-07 | 10.000 | 3.6344e+06 |
| system_14 | LA3 (TP9) | 0.160 | 6.8229e-08 | 10.000 | 3.6887e+05 |
| system_14 | LA3 (TP10) | 0.250 | 6.9439e-09 | 10.000 | 37284 |
| system_14 | LA3 (TP11) | 0.390 | 7.0554e-10 | 10.000 | 3768.7 |
| system_14 | LA3 (TP12) | 0.630 | 7.6625e-11 | 10.000 | 407.83 |
| system_14 | LA3 (TP13) | 0.960 | 1.6641e-11 | 10.000 | 87.117 |
| system_14 | LA3 (TP14) | 1.500 | 1.5774e-11 | 10.000 | 81.805 |
| system_14 | LC3 (TP8) | 0.120 | 6.6269e-07 | 10.000 | 3.6549e+06 |
| system_14 | LC3 (TP9) | 0.180 | 6.8267e-08 | 10.000 | 3.7023e+05 |
| system_14 | LC3 (TP10) | 0.280 | 7.0143e-09 | 10.000 | 37343 |
| system_14 | LC3 (TP11) | 0.440 | 7.0725e-10 | 10.000 | 3774.1 |
| system_14 | LC3 (TP12) | 0.700 | 7.7222e-11 | 10.000 | 410.63 |
| system_14 | LC3 (TP13) | 1.150 | 1.7465e-11 | 10.000 | 91.328 |
| system_14 | LC3 (TP14) | 1.660 | 1.7025e-11 | 10.000 | 88.352 |
| system_14 | Riot (03, 1e-11) | 0m2.181s | 1.0466e-05 | -0.000 | 1.0466e-05 |
| system_14 | Riot (04, 1e-11) | 0m1.239s | 2.1448e-08 | -0.000 | 2.1448e-08 |
| system_14 | Riot (05, 1e-11) | 0m0.348s | 7.1298e-09 | 8.208 | 2.2565e+261 |
| system_14 | Riot (06, 1e-11) | 0m0.194s | 2.2129e-09 | -0.000 | 2.2129e-09 |
| system_14 | Riot (10, 1e-11) | 0m0.126s | 4.0075e-12 | 1.000 | 4.0075e-12 |
| system_14 | Riot (15, 1e-11) | 0m0.175s | 1.2037e-11 | 10.000 | 1.5302e+136 |
| system_14 | Valencia-IVP (0.00025) | 0m1.778s | 0.090273 | 3.670 | 999.58 |
| system_14 | Valencia-IVP (0.0025) | 0m0.165s | 0.90282 | 2.973 | 998.44 |
| system_14 | Valencia-IVP (0.025) | 0m0.021s | 9.1235 | 2.275 | 967.86 |
| system_14 | VNODE-LP (15, 1e-14,1e-14) | 0m0.008s | 1.9185e-13 | 10.000 | 1.0508 |
| system_14 | VNODE-LP (20, 1e-14,1e-14) | 0m0.006s | 2.2737e-13 | 10.000 | 1.25 |
| system_14 | VNODE-LP (25, 1e-14,1e-14) | 0m0.005s | 9.2371e-14 | 10.000 | 0.48828 |

Table 3.11: Simulation results of Problem 15

| Problems | Methods | c5t | c5w | c6t | c6w |
|-----------|----------------------------|----------|---------|--------|---------|
| system_15 | TAYLOR4 (TP8) | 0.110 | 0.9093 | 10.000 | 0.91298 |
| system_15 | TAYLOR4 (TP9) | 0.160 | 0.9093 | 10.000 | 0.91296 |
| system_15 | TAYLOR4 (TP10) | 0.250 | 0.9093 | 10.000 | 0.91296 |
| system_15 | TAYLOR4 (TP11) | 0.410 | 0.9093 | 10.000 | 0.91295 |
| system_15 | TAYLOR4 (TP12) | 0.650 | 0.9093 | 10.000 | 0.91297 |
| system_15 | TAYLOR4 (TP13) | 1.030 | 0.9093 | 10.000 | 0.91297 |
| system_15 | TAYLOR4 (TP14) | 1.590 | 0.9093 | 10.000 | 0.91296 |
| system_15 | RK4 (TP8) | 0.070 | 0.9093 | 10.000 | 0.91299 |
| system_15 | RK4 (TP9) | 0.110 | 0.9093 | 10.000 | 0.91296 |
| system_15 | RK4 (TP10) | 0.180 | 0.9093 | 10.000 | 0.91295 |
| system_15 | RK4 (TP11) | 0.280 | 0.9093 | 10.000 | 0.91296 |
| system_15 | RK4 (TP12) | 0.450 | 0.9093 | 10.000 | 0.91295 |
| system_15 | RK4 (TP13) | 0.710 | 0.9093 | 10.000 | 0.91296 |
| system_15 | RK4 (TP14) | 1.090 | 0.9093 | 10.000 | 0.91295 |
| system_15 | LA3 (TP8) | 0.060 | 1.004 | 10.000 | 41.485 |
| system_15 | LA3 (TP9) | 0.090 | 0.96902 | 10.000 | 25.255 |
| system_15 | LA3 (TP10) | 0.140 | 0.94981 | 10.000 | 9.715 |
| system_15 | LA3 (TP11) | 0.220 | 0.93481 | 10.000 | 6.4485 |
| system_15 | LA3 (TP12) | 0.350 | 0.926 | 10.000 | 3.4445 |
| system_15 | LA3 (TP13) | 0.550 | 0.92025 | 10.000 | 1.6699 |
| system_15 | LA3 (TP14) | 0.870 | 0.91549 | 10.000 | 2.3746 |
| system_15 | LC3 (TP8) | 0.060 | 1.0058 | 10.000 | 63.011 |
| system_15 | LC3 (TP9) | 0.100 | 0.97512 | 10.000 | 22.843 |
| system_15 | LC3 (TP10) | 0.160 | 0.95246 | 10.000 | 16.319 |
| system_15 | LC3 (TP11) | 0.240 | 0.93554 | 10.000 | 8.0286 |
| system_15 | LC3 (TP12) | 0.460 | 0.92607 | 10.000 | 4.1775 |
| system_15 | LC3 (TP13) | 0.620 | 0.92054 | 10.000 | 1.9364 |
| system_15 | LC3 (TP14) | 0.970 | 0.91552 | 10.000 | 1.4643 |
| system_15 | Riot (05, 1e-11) | 0m0.360s | 0.92101 | 10.000 | 0.91295 |
| system_15 | Riot (10, 1e-11) | 0m0.155s | 0.93965 | 10.000 | 0.91295 |
| system_15 | Riot (15, 1e-11) | 0m0.202s | 0.93965 | 10.000 | 0.91295 |
| system_15 | Valencia-IVP (0.00025) | 0m0.976s | 3.6323 | 3.799 | 999.63 |
| system_15 | Valencia-IVP (0.0025) | 0m0.088s | 3.6817 | 3.785 | 999.37 |
| system_15 | Valencia-IVP (0.025) | 0m0.014s | 4.2116 | 3.650 | 997.82 |
| system_15 | VNODE-LP (15, 1e-14,1e-14) | 0m0.004s | 0.9093 | 10.000 | 8.3669 |
| system_15 | VNODE-LP (20, 1e-14,1e-14) | 0m0.006s | 0.9093 | 10.000 | 8.3669 |
| system_15 | VNODE-LP (25, 1e-14,1e-14) | 0m0.003s | 0.9093 | 10.000 | 8.3669 |

Table 3.12: Simulation results of Problem 16

| Problems | Methods | c5t | c5w | c6t | c6w |
|-----------|----------------------------|----------|--------|--------|-------------|
| system_16 | TAYLOR4 (TP8) | 0.190 | 5.0338 | 10.000 | 2.6716e+12 |
| system_16 | TAYLOR4 (TP9) | 0.290 | 5.0338 | 10.000 | 2.6716e+12 |
| system_16 | TAYLOR4 (TP10) | 0.430 | 5.0338 | 10.000 | 2.6716e+12 |
| system_16 | TAYLOR4 (TP11) | 0.660 | 5.0338 | 10.000 | 2.6716e+12 |
| system_16 | TAYLOR4 (TP12) | 1.040 | 5.0338 | 10.000 | 2.6716e+12 |
| system_16 | TAYLOR4 (TP13) | 1.620 | 5.0338 | 10.000 | 2.6716e+12 |
| system_16 | TAYLOR4 (TP14) | 2.530 | 5.0338 | 10.000 | 2.6716e+12 |
| system_16 | RK4 (TP8) | 0.140 | 5.0338 | 10.000 | 2.6716e+12 |
| system_16 | RK4 (TP9) | 0.210 | 5.0338 | 10.000 | 2.6716e+12 |
| system_16 | RK4 (TP10) | 0.330 | 5.0338 | 10.000 | 2.6716e+12 |
| system_16 | RK4 (TP11) | 0.530 | 5.0338 | 10.000 | 2.6716e+12 |
| system_16 | RK4 (TP12) | 0.840 | 5.0338 | 10.000 | 2.6716e+12 |
| system_16 | RK4 (TP13) | 1.270 | 5.0338 | 10.000 | 2.6716e+12 |
| system_16 | RK4 (TP14) | 1.960 | 5.0338 | 10.000 | 2.6716e+12 |
| system_16 | LA3 (TP8) | 0.110 | 5.0368 | 10.000 | 2.6879e+12 |
| system_16 | LA3 (TP9) | 0.170 | 5.035 | 10.000 | 2.678e+12 |
| system_16 | LA3 (TP10) | 0.250 | 5.0343 | 10.000 | 2.6742e+12 |
| system_16 | LA3 (TP11) | 0.410 | 5.034 | 10.000 | 2.6726e+12 |
| system_16 | LA3 (TP12) | 0.670 | 5.0339 | 10.000 | 2.672e+12 |
| system_16 | LA3 (TP13) | 1.030 | 5.0339 | 10.000 | 2.6718e+12 |
| system_16 | LA3 (TP14) | 1.570 | 5.0338 | 10.000 | 2.6717e+12 |
| system_16 | LC3 (TP8) | 0.120 | 5.0391 | 10.000 | 2.7006e+12 |
| system_16 | LC3 (TP9) | 0.190 | 5.0359 | 10.000 | 2.6828e+12 |
| system_16 | LC3 (TP10) | 0.280 | 5.0347 | 10.000 | 2.676e+12 |
| system_16 | LC3 (TP11) | 0.450 | 5.0342 | 10.000 | 2.6734e+12 |
| system_16 | LC3 (TP12) | 0.720 | 5.034 | 10.000 | 2.6723e+12 |
| system_16 | LC3 (TP13) | 1.140 | 5.0339 | 10.000 | 2.6719e+12 |
| system_16 | LC3 (TP14) | 1.740 | 5.0339 | 10.000 | 2.6717e+12 |
| system_16 | Riot (05, 1e-11) | 0m0.607s | 5.0338 | -0.000 | 3.4e+150 |
| system_16 | Riot (10, 1e-11) | 0m0.160s | 5.0338 | -0.000 | 3.3409e+248 |
| system_16 | Riot (15, 1e-11) | 0m0.204s | 5.0338 | -0.000 | 1.3096e+136 |
| system_16 | Valencia-IVP (0.00025) | 0m1.641s | 5.1241 | 2.748 | 999.74 |
| system_16 | Valencia-IVP (0.0025) | 0m0.155s | 5.9373 | 2.635 | 999.64 |
| system_16 | Valencia-IVP (0.025) | 0m0.022s | 14.218 | 2.200 | 938.36 |
| system_16 | VNODE-LP (15, 1e-14,1e-14) | 0m0.004s | 5.0338 | 10.000 | 2.6716e+12 |
| system_16 | VNODE-LP (20, 1e-14,1e-14) | 0m0.004s | 5.0338 | 10.000 | 2.6716e+12 |
| system_16 | VNODE-LP (25, 1e-14,1e-14) | 0m0.005s | 5.0338 | 10.000 | 2.6716e+12 |

Table 3.13: Simulation results of Problem 17

| Problems | Methods | c5t | c5w | c6t | c6w |
|-----------|----------------------------|----------|------------|--------|------------|
| system_17 | TAYLOR4 (TP8) | 0.020 | 2.5429e-08 | 10.000 | 2.3333e-08 |
| system_17 | TAYLOR4 (TP9) | 0.030 | 2.695e-09 | 10.000 | 2.4776e-09 |
| system_17 | TAYLOR4 (TP10) | 0.050 | 2.7876e-10 | 10.000 | 2.6014e-10 |
| system_17 | TAYLOR4 (TP11) | 0.080 | 2.859e-11 | 10.000 | 2.673e-11 |
| system_17 | TAYLOR4 (TP12) | 0.130 | 3.0154e-12 | 10.000 | 2.7828e-12 |
| system_17 | TAYLOR4 (TP13) | 0.200 | 4.6429e-13 | 10.000 | 3.6043e-13 |
| system_17 | TAYLOR4 (TP14) | 0.000 | 0 | 0.000 | 0 |
| system_17 | RK4 (TP8) | 0.010 | 5.9725e-08 | 10.000 | 5.6092e-08 |
| system_17 | RK4 (TP9) | 0.010 | 6.7171e-09 | 10.000 | 6.2806e-09 |
| system_17 | RK4 (TP10) | 0.010 | 6.4465e-10 | 10.000 | 6.282e-10 |
| system_17 | RK4 (TP11) | 0.020 | 5.8932e-11 | 10.000 | 5.8241e-11 |
| system_17 | RK4 (TP12) | 0.040 | 5.3604e-12 | 10.000 | 5.1803e-12 |
| system_17 | RK4 (TP13) | 0.060 | 5.1581e-13 | 10.000 | 4.8617e-13 |
| system_17 | RK4 (TP14) | 0.090 | 8.5709e-14 | 10.000 | 6.3449e-14 |
| system_17 | LA3 (TP8) | 0.010 | 2.395e-08 | 10.000 | 2.1498e-08 |
| system_17 | LA3 (TP9) | 0.010 | 2.5485e-09 | 10.000 | 2.4479e-09 |
| system_17 | LA3 (TP10) | 0.020 | 2.7709e-10 | 10.000 | 2.569e-10 |
| system_17 | LA3 (TP11) | 0.030 | 2.8204e-11 | 10.000 | 2.6542e-11 |
| system_17 | LA3 (TP12) | 0.050 | 2.9106e-12 | 10.000 | 2.7096e-12 |
| system_17 | LA3 (TP13) | 0.080 | 3.2618e-13 | 10.000 | 2.916e-13 |
| system_17 | LA3 (TP14) | 0.130 | 8.2823e-14 | 10.000 | 5.429e-14 |
| system_17 | LC3 (TP8) | 0.010 | 3.2526e-08 | 10.000 | 3.1401e-08 |
| system_17 | LC3 (TP9) | 0.010 | 3.4509e-09 | 10.000 | 3.3385e-09 |
| system_17 | LC3 (TP10) | 0.020 | 3.6045e-10 | 10.000 | 3.4087e-10 |
| system_17 | LC3 (TP11) | 0.030 | 3.4278e-11 | 10.000 | 3.2206e-11 |
| system_17 | LC3 (TP12) | 0.040 | 3.2934e-12 | 10.000 | 3.1542e-12 |
| system_17 | LC3 (TP13) | 0.070 | 3.4661e-13 | 10.000 | 3.1558e-13 |
| system_17 | LC3 (TP14) | 0.110 | 7.2831e-14 | 10.000 | 5.0293e-14 |
| system_17 | Riot (05, 1e-11) | 0m0.209s | 4.0267e-11 | -0.000 | 4.3024e-11 |
| system_17 | Riot (10, 1e-11) | 0m0.153s | 3.8114e-13 | -0.000 | 4.3851e-12 |
| system_17 | Riot (15, 1e-11) | 0m0.249s | 1.7208e-14 | -0.000 | 2.2093e-14 |
| system_17 | Valencia-IVP (0.00025) | 0m1.248s | 0.00062591 | 10.000 | 0.012037 |
| system_17 | Valencia-IVP (0.0025) | 0m0.108s | 0.0062999 | 10.000 | 0.12039 |
| system_17 | Valencia-IVP (0.025) | 0m0.015s | 0.06731 | 9.275 | 1.1674 |
| system_17 | VNODE-LP (15, 1e-14,1e-14) | 0m0.007s | 2.1094e-15 | 10.000 | 1.0825e-15 |
| system_17 | VNODE-LP (20, 1e-14,1e-14) | 0m0.009s | 1.1102e-15 | 10.000 | 9.1593e-16 |
| system_17 | VNODE-LP (25, 1e-14,1e-14) | 0m0.010s | 1.2212e-15 | 10.000 | 5.8287e-16 |

Table 3.14: Simulation results of Problem 18

| Problems | Methods | c5t | c5w | c6t | c6w |
|-----------|----------------------------|-----------|---------|--------|--------|
| system_18 | TAYLOR4 (TP8) | 0.080 | 2.3166 | 1.247 | 80.315 |
| system_18 | TAYLOR4 (TP9) | 0.120 | 2.2033 | 1.271 | 63.866 |
| system_18 | TAYLOR4 (TP10) | 0.190 | 2.136 | 1.286 | 50.824 |
| system_18 | TAYLOR4 (TP11) | 0.280 | 2.0957 | 1.296 | 40.262 |
| system_18 | TAYLOR4 (TP12) | 0.440 | 2.0711 | 1.302 | 31.806 |
| system_18 | TAYLOR4 (TP13) | 0.700 | 2.0558 | 1.305 | 25.183 |
| system_18 | TAYLOR4 (TP14) | 0.000 | 1 | 0.000 | 1 |
| system_18 | RK4 (TP8) | 0.040 | 2.032 | 1.315 | 92.9 |
| system_18 | RK4 (TP9) | 0.060 | 2.031 | 1.315 | 73.775 |
| system_18 | RK4 (TP10) | 0.090 | 2.0305 | 1.315 | 58.317 |
| system_18 | RK4 (TP11) | 0.140 | 2.0303 | 1.315 | 46.315 |
| system_18 | RK4 (TP12) | 0.210 | 2.0303 | 1.314 | 36.66 |
| system_18 | RK4 (TP13) | 0.330 | 2.0302 | 1.313 | 29.062 |
| system_18 | RK4 (TP14) | 0.520 | 2.0302 | 1.312 | 22.972 |
| system_18 | LA3 (TP8) | 0.040 | 2.634 | 1.188 | 103.56 |
| system_18 | LA3 (TP9) | 0.050 | 2.3653 | 1.232 | 82.448 |
| system_18 | LA3 (TP10) | 0.080 | 2.2265 | 1.262 | 64.848 |
| system_18 | LA3 (TP11) | 0.130 | 2.1482 | 1.281 | 51.565 |
| system_18 | LA3 (TP12) | 0.180 | 2.1026 | 1.293 | 40.939 |
| system_18 | LA3 (TP13) | 0.280 | 2.0752 | 1.300 | 32.465 |
| system_18 | LA3 (TP14) | 0.450 | 2.0583 | 1.304 | 25.656 |
| system_18 | LC3 (TP8) | 0.040 | 3.3388 | 1.118 | 99.411 |
| system_18 | LC3 (TP9) | 0.060 | 2.6504 | 1.185 | 79.498 |
| system_18 | LC3 (TP10) | 0.090 | 2.3694 | 1.230 | 63.574 |
| system_18 | LC3 (TP11) | 0.140 | 2.227 | 1.261 | 50.594 |
| system_18 | LC3 (TP12) | 0.200 | 2.1486 | 1.280 | 39.994 |
| system_18 | LC3 (TP13) | 0.310 | 2.1029 | 1.292 | 31.772 |
| system_18 | LC3 (TP14) | 0.490 | 2.0753 | 1.299 | 25.11 |
| system_18 | Riot (05, 1e-11) | 0m3.154s | 0.89498 | -0.000 | 5.6525 |
| system_18 | Riot (10, 1e-11) | 0m12.527s | 0.7695 | -0.000 | 13.258 |
| system_18 | Riot (15, 1e-11) | 0m46.473s | 0.76476 | -0.000 | 12.845 |
| system_18 | Valencia-IVP (0.00025) | 0m3.609s | 2.5351 | 1.309 | 62.299 |
| system_18 | Valencia-IVP (0.0025) | 0m0.385s | 2.4744 | 0.983 | 2.4744 |
| system_18 | Valencia-IVP (0.025) | 0m0.046s | 2.1873 | 0.875 | 2.1873 |
| system_18 | VNODE-LP (15, 1e-14,1e-14) | 0m0.008s | 1.952 | 1.352 | 106.72 |
| system_18 | VNODE-LP (20, 1e-14,1e-14) | 0m0.013s | 4.4163 | 1.079 | 154.57 |
| system_18 | VNODE-LP (25, 1e-14,1e-14) | 0m0.032s | 189.75 | 0.944 | 189.75 |

Table 3.15: Simulation results of Problem 19

| Problems | Methods | c5t | c5w | c6t | c6w |
|-----------|----------------------------|-----------|---------|--------|----------|
| system_19 | TAYLOR4 (TP8) | 0.060 | 0.66694 | 10.000 | 0.18508 |
| system_19 | TAYLOR4 (TP9) | 0.090 | 0.65131 | 10.000 | 0.15696 |
| system_19 | TAYLOR4 (TP10) | 0.130 | 0.64145 | 10.000 | 0.14286 |
| system_19 | TAYLOR4 (TP11) | 0.210 | 0.63542 | 10.000 | 0.13516 |
| system_19 | TAYLOR4 (TP12) | 0.330 | 0.63167 | 10.000 | 0.13071 |
| system_19 | TAYLOR4 (TP13) | 0.520 | 0.62932 | 10.000 | 0.12803 |
| system_19 | TAYLOR4 (TP14) | 0.000 | 1 | 0.000 | 1 |
| system_19 | RK4 (TP8) | 0.030 | 0.62552 | 10.000 | 0.12388 |
| system_19 | RK4 (TP9) | 0.040 | 0.62541 | 10.000 | 0.12377 |
| system_19 | RK4 (TP10) | 0.070 | 0.62536 | 10.000 | 0.12372 |
| system_19 | RK4 (TP11) | 0.100 | 0.62534 | 10.000 | 0.1237 |
| system_19 | RK4 (TP12) | 0.150 | 0.62533 | 10.000 | 0.12369 |
| system_19 | RK4 (TP13) | 0.240 | 0.62533 | 10.000 | 0.12369 |
| system_19 | RK4 (TP14) | 0.380 | 0.62533 | 10.000 | 0.12369 |
| system_19 | LA3 (TP8) | 0.030 | 0.67072 | 10.000 | 0.19253 |
| system_19 | LA3 (TP9) | 0.040 | 0.65354 | 10.000 | 0.15985 |
| system_19 | LA3 (TP10) | 0.060 | 0.64288 | 10.000 | 0.14432 |
| system_19 | LA3 (TP11) | 0.090 | 0.63625 | 10.000 | 0.13591 |
| system_19 | LA3 (TP12) | 0.130 | 0.63216 | 10.000 | 0.13112 |
| system_19 | LA3 (TP13) | 0.210 | 0.62963 | 10.000 | 0.12827 |
| system_19 | LA3 (TP14) | 0.330 | 0.62803 | 10.000 | 0.12654 |
| system_19 | LC3 (TP8) | 0.030 | 0.69287 | 10.000 | 0.25335 |
| system_19 | LC3 (TP9) | 0.040 | 0.66627 | 10.000 | 0.18198 |
| system_19 | LC3 (TP10) | 0.060 | 0.65057 | 10.000 | 0.15488 |
| system_19 | LC3 (TP11) | 0.090 | 0.6409 | 10.000 | 0.14156 |
| system_19 | LC3 (TP12) | 0.140 | 0.63504 | 10.000 | 0.13436 |
| system_19 | LC3 (TP13) | 0.220 | 0.63142 | 10.000 | 0.13021 |
| system_19 | LC3 (TP14) | 0.350 | 0.62915 | 10.000 | 0.12771 |
| system_19 | Riot (05, 1e-11) | 0m3.192s | 0.44827 | -0.000 | 0.13094 |
| system_19 | Riot (10, 1e-11) | 0m12.762s | 0.44389 | -0.000 | 0.057421 |
| system_19 | Riot (15, 1e-11) | 0m40.498s | 0.44387 | -0.000 | 0.055362 |
| system_19 | Valencia-IVP (0.00025) | 0m2.772s | 2.8979 | 1.191 | 3.7768 |
| system_19 | Valencia-IVP (0.0025) | 0m0.287s | 2.9052 | 1.175 | 3.694 |
| system_19 | Valencia-IVP (0.025) | 0m0.041s | 2.9872 | 1.300 | 5.8585 |
| system_19 | VNODE-LP (15, 1e-14,1e-14) | 0m0.008s | 0.88761 | 6.361 | 151.77 |
| system_19 | VNODE-LP (20, 1e-14,1e-14) | 0m0.010s | 0.98714 | 3.815 | 218.19 |
| system_19 | VNODE-LP (25, 1e-14,1e-14) | 0m0.008s | 1.1388 | 2.597 | 270.43 |

Table 3.16: Simulation results of Problem 20

| Problems | Methods | c5t | c5w | c6t | c6w |
|-----------|----------------------------|----------|-----------|--------|------------|
| system_20 | TAYLOR4 (TP8) | 0.040 | 0.0052454 | 10.000 | 5.7321e-09 |
| system_20 | TAYLOR4 (TP9) | 0.060 | 0.0052389 | 10.000 | 5.9775e-10 |
| system_20 | TAYLOR4 (TP10) | 0.100 | 0.005235 | 10.000 | 1.3097e-10 |
| system_20 | TAYLOR4 (TP11) | 0.160 | 0.0052325 | 10.000 | 7.6695e-11 |
| system_20 | TAYLOR4 (TP12) | 0.000 | 0.2 | 0.000 | 0.2 |
| system_20 | TAYLOR4 (TP13) | 0.000 | 0.2 | 0.000 | 0.2 |
| system_20 | TAYLOR4 (TP14) | 0.000 | 0.2 | 0.000 | 0.2 |
| system_20 | RK4 (TP8) | 0.010 | 0.0052285 | 10.000 | 9.8518e-09 |
| system_20 | RK4 (TP9) | 0.020 | 0.0052284 | 10.000 | 1.2709e-09 |
| system_20 | RK4 (TP10) | 0.040 | 0.0052284 | 10.000 | 1.5888e-10 |
| system_20 | RK4 (TP11) | 0.060 | 0.0052284 | 10.000 | 8.1081e-11 |
| system_20 | RK4 (TP12) | 0.100 | 0.0052284 | 10.000 | 6.8557e-11 |
| system_20 | RK4 (TP13) | 0.000 | 0.2 | 0.000 | 0.2 |
| system_20 | RK4 (TP14) | 0.000 | 0.2 | 0.000 | 0.2 |
| system_20 | LA3 (TP8) | 0.010 | 0.0052955 | 10.000 | 2.5286e-07 |
| system_20 | LA3 (TP9) | 0.030 | 0.0052591 | 10.000 | 8.833e-09 |
| system_20 | LA3 (TP10) | 0.040 | 0.0052431 | 10.000 | 8.3868e-10 |
| system_20 | LA3 (TP11) | 0.060 | 0.0052358 | 10.000 | 1.9991e-10 |
| system_20 | LA3 (TP12) | 0.100 | 0.0052323 | 10.000 | 1.02e-10 |
| system_20 | LA3 (TP13) | 0.000 | 0.2 | 0.000 | 0.2 |
| system_20 | LA3 (TP14) | 0.000 | 0.2 | 0.000 | 0.2 |
| system_20 | LC3 (TP8) | 0.010 | 0.0053599 | 10.000 | 9.8946e-07 |
| system_20 | LC3 (TP9) | 0.020 | 0.0052888 | 10.000 | 5.6014e-08 |
| system_20 | LC3 (TP10) | 0.030 | 0.005257 | 10.000 | 4.6691e-09 |
| system_20 | LC3 (TP11) | 0.050 | 0.0052427 | 10.000 | 2.7076e-10 |
| system_20 | LC3 (TP12) | 0.090 | 0.0052359 | 10.000 | 1.1279e-10 |
| system_20 | LC3 (TP13) | 0.140 | 0.0052325 | 10.000 | 8.2115e-11 |
| system_20 | LC3 (TP14) | 0.210 | 0.0052308 | 10.000 | 7.2424e-11 |
| system_20 | Riot (05, 1e-11) | 0m2.343s | 0.0051337 | -0.000 | 6.9818e-11 |
| system_20 | Riot (10, 1e-11) | 0m0.506s | 0.0051337 | -0.000 | 6.6049e-11 |
| system_20 | Riot (15, 1e-11) | 0m1.011s | 0.0051337 | -0.000 | 6.6032e-11 |
| system_20 | Valencia-IVP (0.00025) | 0m2.020s | 5.7609 | 1.371 | 895.46 |
| system_20 | Valencia-IVP (0.0025) | 0m0.244s | 6.1709 | 1.123 | 8.035 |
| system_20 | Valencia-IVP (0.025) | 0m0.030s | 7.1228 | 0.750 | 7.1228 |
| system_20 | VNODE-LP (15, 1e-14,1e-14) | 0m0.003s | 0.0053622 | 10.000 | 6.9172e-11 |
| system_20 | VNODE-LP (20, 1e-14,1e-14) | 0m0.005s | 0.0053887 | 10.000 | 6.957e-11 |
| system_20 | VNODE-LP (25, 1e-14,1e-14) | 0m0.007s | 0.0054356 | 10.000 | 7.0287e-11 |

Table 3.17: Simulation results of Problem 21

| Problems | Methods | c5t | c5w | c6t | c6w |
|-----------|----------------------------|----------|------------|--------|------------|
| system_21 | TAYLOR4 (TP8) | 0.030 | 3.0733e-08 | 10.000 | 6.8721e-09 |
| system_21 | TAYLOR4 (TP9) | 0.050 | 3.2389e-09 | 10.000 | 1.1268e-09 |
| system_21 | TAYLOR4 (TP10) | 0.070 | 3.3001e-10 | 10.000 | 9.6522e-11 |
| system_21 | TAYLOR4 (TP11) | 0.110 | 3.3614e-11 | 10.000 | 8.5003e-12 |
| system_21 | TAYLOR4 (TP12) | 0.000 | 0 | 0.000 | 0 |
| system_21 | TAYLOR4 (TP13) | 0.000 | 0 | 0.000 | 0 |
| system_21 | TAYLOR4 (TP14) | 0.000 | 0 | 0.000 | 0 |
| system_21 | RK4 (TP8) | 0.010 | 3.3937e-08 | 10.000 | 7.4364e-09 |
| system_21 | RK4 (TP9) | 0.020 | 3.4224e-09 | 10.000 | 1.0865e-09 |
| system_21 | RK4 (TP10) | 0.030 | 3.4031e-10 | 10.000 | 7.6861e-11 |
| system_21 | RK4 (TP11) | 0.050 | 3.39e-11 | 10.000 | 1.1213e-11 |
| system_21 | RK4 (TP12) | 0.090 | 3.4204e-12 | 10.000 | 1.3034e-12 |
| system_21 | RK4 (TP13) | 0.000 | 0 | 0.000 | 0 |
| system_21 | RK4 (TP14) | 0.000 | 0 | 0.000 | 0 |
| system_21 | LA3 (TP8) | 0.010 | 2.6881e-08 | 8.634 | 3.8833e-08 |
| system_21 | LA3 (TP9) | 0.020 | 2.8558e-09 | 10.000 | 1.9854e-09 |
| system_21 | LA3 (TP10) | 0.030 | 2.9342e-10 | 10.000 | 1.4172e-10 |
| system_21 | LA3 (TP11) | 0.060 | 2.9966e-11 | 10.000 | 1.0167e-11 |
| system_21 | LA3 (TP12) | 0.090 | 3.0833e-12 | 10.000 | 8.5887e-13 |
| system_21 | LA3 (TP13) | 0.140 | 3.908e-13 | 10.000 | 9.9032e-14 |
| system_21 | LA3 (TP14) | 0.000 | 0 | 0.000 | 0 |
| system_21 | LC3 (TP8) | 0.010 | 3.0304e-08 | 10.000 | 5.0799e-07 |
| system_21 | LC3 (TP9) | 0.020 | 2.7984e-09 | 10.000 | 3.9342e-08 |
| system_21 | LC3 (TP10) | 0.030 | 2.6206e-10 | 10.000 | 2.426e-10 |
| system_21 | LC3 (TP11) | 0.050 | 2.5378e-11 | 10.000 | 1.213e-11 |
| system_21 | LC3 (TP12) | 0.070 | 2.458e-12 | 10.000 | 1.243e-12 |
| system_21 | LC3 (TP13) | 0.120 | 3.082e-13 | 10.000 | 1.0303e-13 |
| system_21 | LC3 (TP14) | 0.190 | 1.39e-13 | 10.000 | 1.6875e-14 |
| system_21 | Riot (05, 1e-11) | 0m0.346s | 4.0035e-11 | -0.000 | 2.075e-12 |
| system_21 | Riot (10, 1e-11) | 0m0.168s | 4.4511e-12 | -0.000 | 7.0832e-14 |
| system_21 | Riot (15, 1e-11) | 0m0.211s | 2.1094e-14 | -0.000 | 2.1094e-14 |
| system_21 | Valencia-IVP (0.00025) | 0m1.174s | 0.073251 | 3.678 | 900.35 |
| system_21 | Valencia-IVP (0.0025) | 0m0.095s | 0.74627 | 2.210 | 6.0933 |
| system_21 | Valencia-IVP (0.025) | 0m0.032s | 6.312 | 0.975 | 6.312 |
| system_21 | VNODE-LP (15, 1e-14,1e-14) | 0m0.008s | 3.9968e-15 | 10.000 | 1.1102e-15 |
| system_21 | VNODE-LP (20, 1e-14,1e-14) | 0m0.007s | 2.8866e-15 | 10.000 | 1.1102e-15 |
| system_21 | VNODE-LP (25, 1e-14,1e-14) | 0m0.006s | 1.9984e-15 | 10.000 | 1.1102e-15 |

Table 3.18: Simulation results of Problem 22

| Problems | Methods | c5t | c5w | c6t | c6w |
|-----------|----------------------------|----------|--------|--------|--------|
| system_22 | TAYLOR4 (TP8) | 0.060 | 1.3818 | 10.000 | 1.3831 |
| system_22 | TAYLOR4 (TP9) | 0.080 | 1.3818 | 10.000 | 1.3831 |
| system_22 | TAYLOR4 (TP10) | 0.120 | 1.3818 | 10.000 | 1.3831 |
| system_22 | TAYLOR4 (TP11) | 0.170 | 1.3818 | 10.000 | 1.3831 |
| system_22 | TAYLOR4 (TP12) | 0.270 | 1.3818 | 10.000 | 1.3831 |
| system_22 | TAYLOR4 (TP13) | 0.440 | 1.3818 | 10.000 | 1.3831 |
| system_22 | TAYLOR4 (TP14) | 0.690 | 1.3818 | 10.000 | 1.3831 |
| system_22 | RK4 (TP8) | 0.040 | 1.3818 | 10.000 | 1.3831 |
| system_22 | RK4 (TP9) | 0.060 | 1.3818 | 10.000 | 1.3831 |
| system_22 | RK4 (TP10) | 0.090 | 1.3818 | 10.000 | 1.3831 |
| system_22 | RK4 (TP11) | 0.130 | 1.3818 | 10.000 | 1.3831 |
| system_22 | RK4 (TP12) | 0.210 | 1.3818 | 10.000 | 1.3831 |
| system_22 | RK4 (TP13) | 0.330 | 1.3818 | 10.000 | 1.3831 |
| system_22 | RK4 (TP14) | 0.520 | 1.3818 | 10.000 | 1.3831 |
| system_22 | LA3 (TP8) | 0.030 | 1.4465 | 10.000 | 5.1497 |
| system_22 | LA3 (TP9) | 0.040 | 1.4248 | 10.000 | 3.046 |
| system_22 | LA3 (TP10) | 0.070 | 1.4096 | 10.000 | 3.5315 |
| system_22 | LA3 (TP11) | 0.100 | 1.4 | 10.000 | 2.4605 |
| system_22 | LA3 (TP12) | 0.170 | 1.3935 | 10.000 | 2.5072 |
| system_22 | LA3 (TP13) | 0.250 | 1.3891 | 10.000 | 1.8036 |
| system_22 | LA3 (TP14) | 0.410 | 1.3865 | 10.000 | 1.5151 |
| system_22 | LC3 (TP8) | 0.040 | 1.4501 | 10.000 | 4.8497 |
| system_22 | LC3 (TP9) | 0.050 | 1.427 | 10.000 | 4.1688 |
| system_22 | LC3 (TP10) | 0.080 | 1.4116 | 10.000 | 2.9464 |
| system_22 | LC3 (TP11) | 0.110 | 1.4004 | 10.000 | 3.0065 |
| system_22 | LC3 (TP12) | 0.180 | 1.394 | 10.000 | 2.0322 |
| system_22 | LC3 (TP13) | 0.290 | 1.3895 | 10.000 | 1.7565 |
| system_22 | LC3 (TP14) | 0.450 | 1.3867 | 10.000 | 1.7305 |
| system_22 | Riot (05, 1e-11) | 0m0.215s | 1.3818 | -0.000 | 1.3831 |
| system_22 | Riot (10, 1e-11) | 0m0.147s | 1.3818 | -0.000 | 1.3831 |
| system_22 | Riot (15, 1e-11) | 0m0.192s | 1.3818 | -0.000 | 1.3831 |
| system_22 | Valencia-IVP (0.00025) | 0m0.980s | 2.7189 | 6.907 | 999.97 |
| system_22 | Valencia-IVP (0.0025) | 0m0.090s | 2.724 | 6.897 | 999.51 |
| system_22 | Valencia-IVP (0.025) | 0m0.014s | 2.7767 | 6.800 | 990.15 |
| system_22 | VNODE-LP (15, 1e-14,1e-14) | 0m0.003s | 1.3818 | 10.000 | 25.373 |
| system_22 | VNODE-LP (20, 1e-14,1e-14) | 0m0.006s | 1.3818 | 10.000 | 25.373 |
| system_22 | VNODE-LP (25, 1e-14,1e-14) | 0m0.005s | 1.3818 | 10.000 | 25.373 |

Table 3.19: Simulation results of Problem 23

| Problems | Methods | c5t | c5w | c6t | c6w |
|-----------|----------------------------|----------|------------|--------|------------|
| system_23 | TAYLOR4 (TP8) | 0.050 | 1.5913e-08 | 10.000 | 1.6814e-06 |
| system_23 | TAYLOR4 (TP9) | 0.080 | 1.7046e-09 | 10.000 | 2.6103e-07 |
| system_23 | TAYLOR4 (TP10) | 0.120 | 1.8306e-10 | 10.000 | 1.6375e-07 |
| system_23 | TAYLOR4 (TP11) | 0.180 | 1.903e-11 | 10.000 | 1.6848e-07 |
| system_23 | TAYLOR4 (TP12) | 0.280 | 2.212e-12 | 10.000 | 2.0574e-08 |
| system_23 | TAYLOR4 (TP13) | 0.450 | 6.5636e-13 | 10.000 | 6.3359e-09 |
| system_23 | TAYLOR4 (TP14) | 0.710 | 7.6073e-13 | 10.000 | 7.5749e-09 |
| system_23 | RK4 (TP8) | 0.040 | 1.9834e-08 | 10.000 | 7.863e-07 |
| system_23 | RK4 (TP9) | 0.050 | 2.2172e-09 | 10.000 | 2.5607e-07 |
| system_23 | RK4 (TP10) | 0.080 | 2.3651e-10 | 10.000 | 8.9245e-08 |
| system_23 | RK4 (TP11) | 0.130 | 2.4555e-11 | 10.000 | 1.3865e-07 |
| system_23 | RK4 (TP12) | 0.200 | 2.6081e-12 | 10.000 | 2.2231e-08 |
| system_23 | RK4 (TP13) | 0.310 | 4.2721e-13 | 10.000 | 3.913e-09 |
| system_23 | RK4 (TP14) | 0.490 | 3.0509e-13 | 10.000 | 2.9406e-09 |
| system_23 | LA3 (TP8) | 0.030 | 1.5086e-08 | 10.000 | 1.2796e-06 |
| system_23 | LA3 (TP9) | 0.040 | 1.6451e-09 | 10.000 | 3.7812e-07 |
| system_23 | LA3 (TP10) | 0.060 | 1.7698e-10 | 10.000 | 1.8245e-07 |
| system_23 | LA3 (TP11) | 0.100 | 1.8517e-11 | 10.000 | 1.1868e-07 |
| system_23 | LA3 (TP12) | 0.150 | 1.9926e-12 | 10.000 | 1.811e-08 |
| system_23 | LA3 (TP13) | 0.240 | 3.233e-13 | 10.000 | 3.106e-09 |
| system_23 | LA3 (TP14) | 0.370 | 2.256e-13 | 10.000 | 2.248e-09 |
| system_23 | LC3 (TP8) | 0.030 | 1.5774e-08 | 10.000 | 1.8135e-06 |
| system_23 | LC3 (TP9) | 0.050 | 1.7152e-09 | 10.000 | 2.948e-07 |
| system_23 | LC3 (TP10) | 0.070 | 1.7917e-10 | 10.000 | 3.0775e-07 |
| system_23 | LC3 (TP11) | 0.110 | 1.8552e-11 | 10.000 | 1.3905e-07 |
| system_23 | LC3 (TP12) | 0.170 | 1.9949e-12 | 10.000 | 1.8803e-08 |
| system_23 | LC3 (TP13) | 0.270 | 3.3529e-13 | 10.000 | 3.2553e-09 |
| system_23 | LC3 (TP14) | 0.420 | 2.4336e-13 | 10.000 | 2.448e-09 |
| system_23 | Riot (05, 1e-11) | 0m0.102s | 5.6269e-11 | -0.000 | 7.3491e-10 |
| system_23 | Riot (10, 1e-11) | 0m0.114s | 2.7978e-14 | -0.000 | 4.2883e-11 |
| system_23 | Riot (15, 1e-11) | 0m0.139s | 4.1966e-14 | -0.000 | 1.0757e-12 |
| system_23 | Valencia-IVP (0.00025) | 0m1.130s | 0.00046233 | 10.000 | 5.0012 |
| system_23 | Valencia-IVP (0.0025) | 0m0.095s | 0.0046322 | 10.000 | 50.642 |
| system_23 | Valencia-IVP (0.025) | 0m0.014s | 0.047235 | 10.000 | 574.87 |
| system_23 | VNODE-LP (15, 1e-14,1e-14) | 0m0.003s | 1.9984e-15 | 10.000 | 8.3933e-14 |
| system_23 | VNODE-LP (20, 1e-14,1e-14) | 0m0.004s | 9.992e-16 | 10.000 | 7.5051e-14 |
| system_23 | VNODE-LP (25, 1e-14,1e-14) | 0m0.006s | 6.6613e-16 | 10.000 | 4.7073e-14 |

Table 3.20: Simulation results of Problem 24

| Problems | Methods | c5t | c5w | c6t | c6w |
|-----------|----------------------------|----------|--------|--------|--------|
| system_24 | TAYLOR4 (TP8) | 0.060 | 1.9324 | 10.000 | 14317 |
| system_24 | TAYLOR4 (TP9) | 0.080 | 1.9324 | 10.000 | 14317 |
| system_24 | TAYLOR4 (TP10) | 0.110 | 1.9324 | 10.000 | 14317 |
| system_24 | TAYLOR4 (TP11) | 0.170 | 1.9324 | 10.000 | 14317 |
| system_24 | TAYLOR4 (TP12) | 0.270 | 1.9324 | 10.000 | 14317 |
| system_24 | TAYLOR4 (TP13) | 0.440 | 1.9324 | 10.000 | 14317 |
| system_24 | TAYLOR4 (TP14) | 0.670 | 1.9324 | 10.000 | 14317 |
| system_24 | RK4 (TP8) | 0.040 | 1.9324 | 10.000 | 14317 |
| system_24 | RK4 (TP9) | 0.060 | 1.9324 | 10.000 | 14317 |
| system_24 | RK4 (TP10) | 0.080 | 1.9324 | 10.000 | 14317 |
| system_24 | RK4 (TP11) | 0.130 | 1.9324 | 10.000 | 14317 |
| system_24 | RK4 (TP12) | 0.200 | 1.9324 | 10.000 | 14317 |
| system_24 | RK4 (TP13) | 0.330 | 1.9324 | 10.000 | 14317 |
| system_24 | RK4 (TP14) | 0.510 | 1.9324 | 10.000 | 14317 |
| system_24 | LA3 (TP8) | 0.030 | 1.9328 | 10.000 | 14347 |
| system_24 | LA3 (TP9) | 0.050 | 1.9326 | 10.000 | 14329 |
| system_24 | LA3 (TP10) | 0.070 | 1.9325 | 10.000 | 14322 |
| system_24 | LA3 (TP11) | 0.100 | 1.9325 | 10.000 | 14319 |
| system_24 | LA3 (TP12) | 0.160 | 1.9324 | 10.000 | 14318 |
| system_24 | LA3 (TP13) | 0.250 | 1.9324 | 10.000 | 14318 |
| system_24 | LA3 (TP14) | 0.400 | 1.9324 | 10.000 | 14317 |
| system_24 | LC3 (TP8) | 0.040 | 1.9331 | 10.000 | 14371 |
| system_24 | LC3 (TP9) | 0.050 | 1.9327 | 10.000 | 14338 |
| system_24 | LC3 (TP10) | 0.070 | 1.9325 | 10.000 | 14325 |
| system_24 | LC3 (TP11) | 0.110 | 1.9325 | 10.000 | 14320 |
| system_24 | LC3 (TP12) | 0.180 | 1.9325 | 10.000 | 14318 |
| system_24 | LC3 (TP13) | 0.320 | 1.9324 | 10.000 | 14318 |
| system_24 | LC3 (TP14) | 0.470 | 1.9324 | 10.000 | 14317 |
| system_24 | Riot (05, 1e-11) | 0m0.222s | 1.9324 | -0.000 | 21721 |
| system_24 | Riot (10, 1e-11) | 0m0.148s | 1.9324 | -0.000 | 21718 |
| system_24 | Riot (15, 1e-11) | 0m0.193s | 1.9324 | -0.000 | 21703 |
| system_24 | Valencia-IVP (0.00025) | 0m1.214s | 1.9329 | 7.337 | 999.94 |
| system_24 | Valencia-IVP (0.0025) | 0m0.114s | 1.9368 | 7.320 | 998.62 |
| system_24 | Valencia-IVP (0.025) | 0m0.014s | 1.977 | 7.175 | 998.34 |
| system_24 | VNODE-LP (15, 1e-14,1e-14) | 0m0.004s | 1.9324 | 10.000 | 14317 |
| system_24 | VNODE-LP (20, 1e-14,1e-14) | 0m0.006s | 1.9324 | 10.000 | 14317 |
| system_24 | VNODE-LP (25, 1e-14,1e-14) | 0m0.002s | 1.9324 | 10.000 | 14317 |

Table 3.21: Simulation results of Problem 25

| Problems | Methods | c5t | c5w | c6t | c6w |
|-----------|----------------------------|----------|------------|--------|------------|
| system_25 | TAYLOR4 (TP8) | 0.040 | 1.3772e-08 | 10.000 | 0.00016615 |
| system_25 | TAYLOR4 (TP9) | 0.070 | 1.5056e-09 | 10.000 | 1.7744e-05 |
| system_25 | TAYLOR4 (TP10) | 0.110 | 1.6064e-10 | 10.000 | 1.8561e-06 |
| system_25 | TAYLOR4 (TP11) | 0.160 | 1.6748e-11 | 10.000 | 1.9164e-07 |
| system_25 | TAYLOR4 (TP12) | 0.260 | 1.8416e-12 | 10.000 | 2.0722e-08 |
| system_25 | TAYLOR4 (TP13) | 0.410 | 4.1234e-13 | 10.000 | 4.1866e-09 |
| system_25 | TAYLOR4 (TP14) | 0.650 | 4.0101e-13 | 10.000 | 3.7742e-09 |
| system_25 | RK4 (TP8) | 0.030 | 1.7587e-08 | 10.000 | 0.00021528 |
| system_25 | RK4 (TP9) | 0.050 | 1.9552e-09 | 10.000 | 2.3157e-05 |
| system_25 | RK4 (TP10) | 0.070 | 2.0936e-10 | 10.000 | 2.4276e-06 |
| system_25 | RK4 (TP11) | 0.110 | 2.1817e-11 | 10.000 | 2.501e-07 |
| system_25 | RK4 (TP12) | 0.180 | 2.26e-12 | 10.000 | 2.5778e-08 |
| system_25 | RK4 (TP13) | 0.280 | 3.1397e-13 | 10.000 | 3.3969e-09 |
| system_25 | RK4 (TP14) | 0.450 | 1.6809e-13 | 10.000 | 1.6121e-09 |
| system_25 | LA3 (TP8) | 0.030 | 1.2714e-08 | 10.000 | 0.00016167 |
| system_25 | LA3 (TP9) | 0.040 | 1.4237e-09 | 10.000 | 1.7264e-05 |
| system_25 | LA3 (TP10) | 0.060 | 1.537e-10 | 10.000 | 1.7994e-06 |
| system_25 | LA3 (TP11) | 0.090 | 1.61e-11 | 10.000 | 1.8681e-07 |
| system_25 | LA3 (TP12) | 0.140 | 1.714e-12 | 10.000 | 1.9526e-08 |
| system_25 | LA3 (TP13) | 0.210 | 2.3848e-13 | 10.000 | 2.5806e-09 |
| system_25 | LA3 (TP14) | 0.340 | 1.2601e-13 | 10.000 | 1.2041e-09 |
| system_25 | LC3 (TP8) | 0.030 | 1.2989e-08 | 10.000 | 0.00016663 |
| system_25 | LC3 (TP9) | 0.040 | 1.473e-09 | 10.000 | 1.7777e-05 |
| system_25 | LC3 (TP10) | 0.060 | 1.562e-10 | 10.000 | 1.8255e-06 |
| system_25 | LC3 (TP11) | 0.100 | 1.6272e-11 | 10.000 | 1.878e-07 |
| system_25 | LC3 (TP12) | 0.150 | 1.7181e-12 | 10.000 | 1.9601e-08 |
| system_25 | LC3 (TP13) | 0.240 | 2.4358e-13 | 10.000 | 2.6186e-09 |
| system_25 | LC3 (TP14) | 0.380 | 1.3412e-13 | 10.000 | 1.2862e-09 |
| system_25 | Riot (05, 1e-11) | 0m0.104s | 5.7086e-11 | -0.000 | 0.0013639 |
| system_25 | Riot (10, 1e-11) | 0m0.109s | 3.7192e-15 | -0.000 | 3.7192e-15 |
| system_25 | Riot (15, 1e-11) | 0m0.089s | 0 | -0.000 | 5.7732e-15 |
| system_25 | Valencia-IVP (0.00025) | 0m1.087s | 0.00029389 | 10.000 | 2.7571 |
| system_25 | Valencia-IVP (0.0025) | 0m0.093s | 0.0029465 | 10.000 | 27.915 |
| system_25 | Valencia-IVP (0.025) | 0m0.015s | 0.030251 | 10.000 | 316.61 |
| system_25 | VNODE-LP (15, 1e-14,1e-14) | 0m0.004s | 9.992e-16 | 10.000 | 8.9433e-12 |
| system_25 | VNODE-LP (20, 1e-14,1e-14) | 0m0.004s | 8.8818e-16 | 10.000 | 7.9496e-12 |
| system_25 | VNODE-LP (25, 1e-14,1e-14) | 0m0.004s | 8.3267e-16 | 10.000 | 6.2134e-12 |

Table 3.22: Simulation results of Problem 26

| Problems | Methods | c5t | c5w | c6t | c6w |
|-----------|----------------------------|----------|--------|--------|------------|
| system_26 | TAYLOR4 (TP8) | 0.190 | 1.2981 | 10.000 | 0.00023241 |
| system_26 | TAYLOR4 (TP9) | 0.280 | 1.2981 | 10.000 | 0.00022813 |
| system_26 | TAYLOR4 (TP10) | 0.410 | 1.2981 | 10.000 | 0.00022749 |
| system_26 | TAYLOR4 (TP11) | 0.610 | 1.2981 | 10.000 | 0.000229 |
| system_26 | TAYLOR4 (TP12) | 0.940 | 1.2981 | 10.000 | 0.00022964 |
| system_26 | TAYLOR4 (TP13) | 1.480 | 1.2981 | 10.000 | 0.00022949 |
| system_26 | TAYLOR4 (TP14) | 2.360 | 1.2981 | 10.000 | 0.00022948 |
| system_26 | RK4 (TP8) | 0.130 | 1.2981 | 10.000 | 0.00023297 |
| system_26 | RK4 (TP9) | 0.190 | 1.2981 | 10.000 | 0.00022751 |
| system_26 | RK4 (TP10) | 0.300 | 1.2981 | 10.000 | 0.00022782 |
| system_26 | RK4 (TP11) | 0.470 | 1.2981 | 10.000 | 0.00022747 |
| system_26 | RK4 (TP12) | 0.730 | 1.2981 | 10.000 | 0.00022897 |
| system_26 | RK4 (TP13) | 1.150 | 1.2981 | 10.000 | 0.00022902 |
| system_26 | RK4 (TP14) | 1.800 | 1.2981 | 10.000 | 0.00022939 |
| system_26 | LA3 (TP8) | 0.110 | 1.7614 | 10.000 | 47.327 |
| system_26 | LA3 (TP9) | 0.160 | 1.6099 | 10.000 | 5.1636 |
| system_26 | LA3 (TP10) | 0.240 | 1.5196 | 10.000 | 3.3388 |
| system_26 | LA3 (TP11) | 0.380 | 1.4341 | 10.000 | 1.4904 |
| system_26 | LA3 (TP12) | 0.580 | 1.394 | 10.000 | 0.77395 |
| system_26 | LA3 (TP13) | 0.910 | 1.3539 | 10.000 | 0.27453 |
| system_26 | LA3 (TP14) | 1.440 | 1.3298 | 10.000 | 0.09762 |
| system_26 | LC3 (TP8) | 0.110 | 1.7962 | 10.000 | 57.572 |
| system_26 | LC3 (TP9) | 0.170 | 1.6345 | 10.000 | 4.2967 |
| system_26 | LC3 (TP10) | 0.260 | 1.544 | 10.000 | 3.6973 |
| system_26 | LC3 (TP11) | 0.420 | 1.4629 | 10.000 | 3.522 |
| system_26 | LC3 (TP12) | 0.630 | 1.3763 | 10.000 | 0.85036 |
| system_26 | LC3 (TP13) | 1.000 | 1.3639 | 10.000 | 1.2933 |
| system_26 | LC3 (TP14) | 1.600 | 1.3283 | 10.000 | 0.13024 |
| system_26 | Riot (05, 1e-11) | 0m0.592s | 1.2981 | -0.000 | 0.00023441 |
| system_26 | Riot (10, 1e-11) | 0m0.217s | 1.2981 | -0.000 | 0.00022716 |
| system_26 | Riot (15, 1e-11) | 0m0.302s | 1.2981 | -0.000 | 0.00022731 |
| system_26 | Valencia-IVP (0.00025) | 0m1.817s | 277.25 | 1.238 | 999.84 |
| system_26 | Valencia-IVP (0.0025) | 0m0.156s | 287.15 | 1.230 | 996.77 |
| system_26 | Valencia-IVP (0.025) | 0m0.022s | 421.64 | 1.125 | 867.43 |
| system_26 | VNODE-LP (15, 1e-14,1e-14) | 0m0.007s | 1.2981 | 10.000 | 6.8883 |
| system_26 | VNODE-LP (20, 1e-14,1e-14) | 0m0.008s | 1.2981 | 10.000 | 6.8883 |
| system_26 | VNODE-LP (25, 1e-14,1e-14) | 0m0.007s | 1.2981 | 10.000 | 6.8883 |

Table 3.23: Simulation results of Problem 27

| Problems | Methods | c5t | c5w | c6t | c6w |
|-----------|----------------------------|----------|------------|--------|------------|
| system_27 | TAYLOR4 (TP8) | 0.150 | 9.9382e-08 | 10.000 | 9.9453e-07 |
| system_27 | TAYLOR4 (TP9) | 0.210 | 1.0984e-08 | 10.000 | 2.4889e-07 |
| system_27 | TAYLOR4 (TP10) | 0.320 | 1.1848e-09 | 10.000 | 3.0464e-07 |
| system_27 | TAYLOR4 (TP11) | 0.500 | 1.2016e-10 | 10.000 | 2.585e-07 |
| system_27 | TAYLOR4 (TP12) | 0.790 | 1.366e-11 | 10.000 | 2.1613e-07 |
| system_27 | TAYLOR4 (TP13) | 1.220 | 3.6535e-12 | 10.000 | 3.5367e-07 |
| system_27 | TAYLOR4 (TP14) | 1.900 | 3.9741e-12 | 10.000 | 1.2891e-06 |
| system_27 | RK4 (TP8) | 0.110 | 1.2932e-07 | 10.000 | 1.3248e-06 |
| system_27 | RK4 (TP9) | 0.160 | 1.4163e-08 | 10.000 | 2.0043e-07 |
| system_27 | RK4 (TP10) | 0.250 | 1.5098e-09 | 10.000 | 1.5059e-07 |
| system_27 | RK4 (TP11) | 0.410 | 1.5404e-10 | 10.000 | 1.5609e-07 |
| system_27 | RK4 (TP12) | 0.640 | 1.6365e-11 | 10.000 | 1.7223e-07 |
| system_27 | RK4 (TP13) | 0.970 | 2.5304e-12 | 10.000 | 1.8389e-07 |
| system_27 | RK4 (TP14) | 1.540 | 1.6289e-12 | 10.000 | 1.7725e-07 |
| system_27 | LA3 (TP8) | 0.090 | 1.077e-07 | 10.000 | 0.0021861 |
| system_27 | LA3 (TP9) | 0.130 | 1.1278e-08 | 10.000 | 0.00013224 |
| system_27 | LA3 (TP10) | 0.200 | 1.1811e-09 | 10.000 | 3.2675e-05 |
| system_27 | LA3 (TP11) | 0.310 | 1.2139e-10 | 10.000 | 3.5598e-06 |
| system_27 | LA3 (TP12) | 0.490 | 1.2736e-11 | 10.000 | 1.2922e-06 |
| system_27 | LA3 (TP13) | 0.760 | 1.9278e-12 | 10.000 | 3.4537e-07 |
| system_27 | LA3 (TP14) | 1.230 | 1.2119e-12 | 10.000 | 3.651e-07 |
| system_27 | LC3 (TP8) | 0.100 | 1.1371e-07 | 10.000 | 0.0045486 |
| system_27 | LC3 (TP9) | 0.150 | 1.1741e-08 | 10.000 | 0.0004441 |
| system_27 | LC3 (TP10) | 0.220 | 1.2156e-09 | 10.000 | 4.9058e-05 |
| system_27 | LC3 (TP11) | 0.350 | 1.2311e-10 | 10.000 | 4.7915e-06 |
| system_27 | LC3 (TP12) | 0.540 | 1.297e-11 | 10.000 | 1.3287e-06 |
| system_27 | LC3 (TP13) | 0.840 | 1.9971e-12 | 10.000 | 4.0411e-07 |
| system_27 | LC3 (TP14) | 1.290 | 1.3069e-12 | 10.000 | 1.3527e-06 |
| system_27 | Riot (05, 1e-11) | 0m0.256s | 1.8868e-10 | -0.000 | 2.7813e+09 |
| system_27 | Riot (10, 1e-11) | 0m0.164s | 1.199e-14 | -0.000 | 3.4514e-08 |
| system_27 | Riot (15, 1e-11) | 0m0.230s | 8.793e-14 | -0.000 | 1.8045e-12 |
| system_27 | Valencia-IVP (0.00025) | 0m1.391s | 0.1407 | 2.649 | 999.19 |
| system_27 | Valencia-IVP (0.0025) | 0m0.126s | 1.4595 | 2.205 | 988.39 |
| system_27 | Valencia-IVP (0.025) | 0m0.021s | 21.761 | 1.650 | 925.46 |
| system_27 | VNODE-LP (15, 1e-14,1e-14) | 0m0.006s | 9.992e-15 | 10.000 | 9.4229e-14 |
| system_27 | VNODE-LP (20, 1e-14,1e-14) | 0m0.005s | 5.9952e-15 | 10.000 | 5.4546e-14 |
| system_27 | VNODE-LP (25, 1e-14,1e-14) | 0m0.004s | 5.9952e-15 | 10.000 | 3.6599e-14 |

Table 3.24: Simulation results of Problem 28

| Problems | Methods | c5t | c5w | c6t | c6w |
|-----------|----------------------------|------------|---------|--------|---------|
| system_28 | TAYLOR4 (TP8) | 4.130 | 20.932 | 0.283 | 20.932 |
| system_28 | TAYLOR4 (TP9) | 5.960 | 17.968 | 0.282 | 17.968 |
| system_28 | TAYLOR4 (TP10) | 8.860 | 15.338 | 0.280 | 15.338 |
| system_28 | TAYLOR4 (TP11) | 12.540 | 13.052 | 0.282 | 13.052 |
| system_28 | TAYLOR4 (TP12) | 18.350 | 11.079 | 0.280 | 11.079 |
| system_28 | TAYLOR4 (TP13) | 27.080 | 9.385 | 0.277 | 9.385 |
| system_28 | TAYLOR4 (TP14) | 38.040 | 7.9094 | 0.275 | 7.9094 |
| system_28 | RK4 (TP8) | 4.160 | 23.152 | 0.284 | 23.152 |
| system_28 | RK4 (TP9) | 3.300 | 19.86 | 0.284 | 19.86 |
| system_28 | RK4 (TP10) | 4.790 | 17.013 | 0.285 | 17.013 |
| system_28 | RK4 (TP11) | 6.890 | 14.506 | 0.283 | 14.506 |
| system_28 | RK4 (TP12) | 9.590 | 12.365 | 0.279 | 12.365 |
| system_28 | RK4 (TP13) | 13.920 | 10.475 | 0.278 | 10.475 |
| system_28 | RK4 (TP14) | 20.070 | 8.8608 | 0.276 | 8.8608 |
| system_28 | LA3 (TP8) | 1.960 | 24.682 | 0.274 | 24.682 |
| system_28 | LA3 (TP9) | 2.960 | 21.217 | 0.276 | 21.217 |
| system_28 | LA3 (TP10) | 4.250 | 18.246 | 0.276 | 18.246 |
| system_28 | LA3 (TP11) | 6.350 | 15.618 | 0.278 | 15.618 |
| system_28 | LA3 (TP12) | 8.760 | 13.281 | 0.278 | 13.281 |
| system_28 | LA3 (TP13) | 12.720 | 11.269 | 0.277 | 11.269 |
| system_28 | LA3 (TP14) | 18.360 | 9.5482 | 0.276 | 9.5482 |
| system_28 | LC3 (TP8) | 1.870 | 24.835 | 0.265 | 24.835 |
| system_28 | LC3 (TP9) | 2.830 | 21.005 | 0.270 | 21.005 |
| system_28 | LC3 (TP10) | 4.250 | 17.966 | 0.273 | 17.966 |
| system_28 | LC3 (TP11) | 6.320 | 15.404 | 0.276 | 15.404 |
| system_28 | LC3 (TP12) | 8.960 | 13.092 | 0.276 | 13.092 |
| system_28 | LC3 (TP13) | 13.090 | 11.123 | 0.277 | 11.123 |
| system_28 | LC3 (TP14) | 19.040 | 9.4039 | 0.275 | 9.4039 |
| system_28 | Riot (05, 1e-11) | 0m29.200s | 0 | -0.000 | 4.2446 |
| system_28 | Riot (10, 1e-11) | 18m44.691s | 0 | -0.000 | 4.0786 |
| system_28 | Riot (15, 1e-11) | 210m1.595s | 0 | -0.000 | 4.5904 |
| system_28 | Valencia-IVP (0.00025) | 0m2.126s | 1.1713 | 0.162 | 1.1713 |
| system_28 | Valencia-IVP (0.0025) | 0m0.733s | 3.1672 | 0.395 | 3.1672 |
| system_28 | Valencia-IVP (0.025) | 0m0.027s | 0.95755 | 0.075 | 0.95755 |
| system_28 | VNODE-LP (15, 1e-14,1e-14) | 0m0.309s | 18.119 | 0.155 | 18.119 |
| system_28 | VNODE-LP (20, 1e-14,1e-14) | 0m0.299s | 22.402 | 0.140 | 22.402 |
| system_28 | VNODE-LP (25, 1e-14,1e-14) | 0m0.301s | 25.252 | 0.128 | 25.252 |

Table 3.25: Simulation results of Problem 29

| Problems | Methods | c5t | c5w | c6t | c6w |
|-----------|----------------------------|----------|------------|--------|------------|
| system_29 | TAYLOR4 (TP8) | 0.560 | 3.5522e-07 | 10.000 | 3.7763e-07 |
| system_29 | TAYLOR4 (TP9) | 0.830 | 3.6736e-08 | 10.000 | 3.9348e-08 |
| system_29 | TAYLOR4 (TP10) | 1.250 | 3.7576e-09 | 10.000 | 4.0256e-09 |
| system_29 | TAYLOR4 (TP11) | 1.990 | 3.7343e-10 | 10.000 | 4.03e-10 |
| system_29 | TAYLOR4 (TP12) | 3.120 | 3.7579e-11 | 10.000 | 4.0723e-11 |
| system_29 | TAYLOR4 (TP13) | 5.370 | 4.5068e-12 | 10.000 | 4.8452e-12 |
| system_29 | TAYLOR4 (TP14) | 7.360 | 1.6607e-12 | 10.000 | 1.7164e-12 |
| system_29 | RK4 (TP8) | 0.340 | 5.0539e-07 | 10.000 | 5.8051e-07 |
| system_29 | RK4 (TP9) | 0.460 | 5.8113e-08 | 10.000 | 6.5063e-08 |
| system_29 | RK4 (TP10) | 0.710 | 5.7374e-09 | 10.000 | 6.4068e-09 |
| system_29 | RK4 (TP11) | 1.000 | 6.2862e-10 | 10.000 | 6.6449e-10 |
| system_29 | RK4 (TP12) | 1.540 | 6.1003e-11 | 10.000 | 6.3473e-11 |
| system_29 | RK4 (TP13) | 2.420 | 5.9718e-12 | 10.000 | 6.1513e-12 |
| system_29 | RK4 (TP14) | 3.700 | 8.6475e-13 | 10.000 | 8.7147e-13 |
| system_29 | LA3 (TP8) | 0.340 | 2.3268e-07 | 10.000 | 3.1546e-07 |
| system_29 | LA3 (TP9) | 0.470 | 2.4512e-08 | 10.000 | 3.3829e-08 |
| system_29 | LA3 (TP10) | 0.710 | 2.3962e-09 | 10.000 | 3.3821e-09 |
| system_29 | LA3 (TP11) | 1.060 | 2.281e-10 | 10.000 | 3.2909e-10 |
| system_29 | LA3 (TP12) | 1.660 | 2.1874e-11 | 10.000 | 3.1581e-11 |
| system_29 | LA3 (TP13) | 2.600 | 2.3522e-12 | 10.000 | 3.2499e-12 |
| system_29 | LA3 (TP14) | 4.030 | 5.4412e-13 | 10.000 | 6.1251e-13 |
| system_29 | LC3 (TP8) | 0.340 | 2.9012e-07 | 10.000 | 4.1042e-07 |
| system_29 | LC3 (TP9) | 0.460 | 2.9152e-08 | 10.000 | 4.3464e-08 |
| system_29 | LC3 (TP10) | 0.680 | 2.7738e-09 | 10.000 | 4.2533e-09 |
| system_29 | LC3 (TP11) | 1.000 | 2.5993e-10 | 10.000 | 3.9511e-10 |
| system_29 | LC3 (TP12) | 1.560 | 2.3858e-11 | 10.000 | 3.5866e-11 |
| system_29 | LC3 (TP13) | 2.480 | 2.4346e-12 | 10.000 | 3.4715e-12 |
| system_29 | LC3 (TP14) | 3.730 | 5.4146e-13 | 10.000 | 6.1394e-13 |
| system_29 | Riot (05, 1e-11) | 0m1.818s | 3.2308e-10 | -0.000 | 5.7962e-09 |
| system_29 | Riot (10, 1e-11) | 0m1.333s | 6.1563e-12 | -0.000 | 1.0335e-10 |
| system_29 | Riot (15, 1e-11) | 0m2.386s | 9.6034e-15 | -0.000 | 9.6034e-15 |
| system_29 | Valencia-IVP (0.00025) | 0m3.140s | 0.001153 | 10.000 | 0.057922 |
| system_29 | Valencia-IVP (0.0025) | 0m0.516s | 0.01199 | 6.265 | 0.2962 |
| system_29 | Valencia-IVP (0.025) | 0m0.226s | 0.17131 | 1.200 | 0.2357 |
| system_29 | VNODE-LP (15, 1e-14,1e-14) | 0m0.080s | 1.8485e-14 | 10.000 | 1.5952e-14 |
| system_29 | VNODE-LP (20, 1e-14,1e-14) | 0m0.099s | 1.199e-14 | 10.000 | 1.1606e-14 |
| system_29 | VNODE-LP (25, 1e-14,1e-14) | 0m0.107s | 9.4924e-15 | 10.000 | 8.9239e-15 |

Table 3.26: Simulation results of Problem 30

| Problems | Methods | c5t | c5w | c6t | c6w |
|-----------|----------------------------|-----------|--------|-------|--------|
| system_30 | TAYLOR4 (TP8) | 16.140 | 80.278 | 0.297 | 80.278 |
| system_30 | TAYLOR4 (TP9) | 24.390 | 63.718 | 0.298 | 63.718 |
| system_30 | TAYLOR4 (TP10) | 36.140 | 50.476 | 0.299 | 50.476 |
| system_30 | TAYLOR4 (TP11) | 53.200 | 40.071 | 0.299 | 40.071 |
| system_30 | TAYLOR4 (TP12) | 77.710 | 31.765 | 0.299 | 31.765 |
| system_30 | TAYLOR4 (TP13) | 113.360 | 25.075 | 0.297 | 25.075 |
| system_30 | TAYLOR4 (TP14) | 159.450 | 19.754 | 0.296 | 19.754 |
| system_30 | RK4 (TP8) | 9.090 | 92.703 | 0.302 | 92.703 |
| system_30 | RK4 (TP9) | 13.060 | 73.155 | 0.302 | 73.155 |
| system_30 | RK4 (TP10) | 19.520 | 58.308 | 0.303 | 58.308 |
| system_30 | RK4 (TP11) | 28.190 | 46.348 | 0.300 | 46.348 |
| system_30 | RK4 (TP12) | 40.790 | 36.606 | 0.300 | 36.606 |
| system_30 | RK4 (TP13) | 60.340 | 28.938 | 0.299 | 28.938 |
| system_30 | RK4 (TP14) | 87.210 | 22.85 | 0.298 | 22.85 |
| system_30 | LA3 (TP8) | 7.670 | 103.64 | 0.290 | 103.64 |
| system_30 | LA3 (TP9) | 11.260 | 81.407 | 0.293 | 81.407 |
| system_30 | LA3 (TP10) | 16.760 | 65.057 | 0.296 | 65.057 |
| system_30 | LA3 (TP11) | 24.890 | 51.523 | 0.298 | 51.523 |
| system_30 | LA3 (TP12) | 36.280 | 40.863 | 0.298 | 40.863 |
| system_30 | LA3 (TP13) | 55.410 | 32.308 | 0.298 | 32.308 |
| system_30 | LA3 (TP14) | 82.570 | 25.57 | 0.297 | 25.57 |
| system_30 | LC3 (TP8) | 7.590 | 101.33 | 0.282 | 101.33 |
| system_30 | LC3 (TP9) | 11.350 | 79.317 | 0.288 | 79.317 |
| system_30 | LC3 (TP10) | 17.050 | 63.817 | 0.293 | 63.817 |
| system_30 | LC3 (TP11) | 25.100 | 50.599 | 0.296 | 50.599 |
| system_30 | LC3 (TP12) | 36.770 | 40.033 | 0.296 | 40.033 |
| system_30 | LC3 (TP13) | 55.480 | 31.626 | 0.297 | 31.626 |
| system_30 | LC3 (TP14) | 78.840 | 25.014 | 0.297 | 25.014 |
| system_30 | Riot | | | | |
| system_30 | Valencia-IVP (0.00025) | 0m13.555s | 57.455 | 0.332 | 57.455 |
| system_30 | Valencia-IVP (0.0025) | 0m0.494s | 4.4295 | 0.245 | 4.4295 |
| system_30 | Valencia-IVP (0.025) | 0m0.108s | 3.7929 | 0.200 | 3.7929 |
| system_30 | VNODE-LP (15, 1e-14,1e-14) | 0m0.194s | 105.32 | 0.259 | 105.32 |
| system_30 | VNODE-LP (20, 1e-14,1e-14) | 0m0.186s | 146.87 | 0.237 | 146.87 |
| system_30 | VNODE-LP (25, 1e-14,1e-14) | 0m0.187s | 188.72 | 0.220 | 188.72 |

Table 3.27: Simulation results of Problem 31

| Problems | Methods | c5t | c5w | c6t | c6w |
|-----------|----------------------------|-----------|------------|--------|------------|
| system_31 | TAYLOR4 (TP8) | 2.230 | 1.2578e-07 | 10.000 | 5.8598e-05 |
| system_31 | TAYLOR4 (TP9) | 3.470 | 1.3976e-08 | 10.000 | 5.2745e-05 |
| system_31 | TAYLOR4 (TP10) | 5.330 | 1.5217e-09 | 10.000 | 1.2559e-05 |
| system_31 | TAYLOR4 (TP11) | 8.330 | 1.5944e-10 | 10.000 | 1.4182e-06 |
| system_31 | TAYLOR4 (TP12) | 13.050 | 1.6868e-11 | 10.000 | 1.5655e-07 |
| system_31 | TAYLOR4 (TP13) | 20.400 | 2.2326e-12 | 10.000 | 2.1442e-08 |
| system_31 | TAYLOR4 (TP14) | 31.840 | 1.028e-12 | 10.000 | 1.025e-08 |
| system_31 | RK4 (TP8) | 1.290 | 1.0486e-07 | 10.000 | 4.4015e-05 |
| system_31 | RK4 (TP9) | 1.850 | 1.2451e-08 | 10.000 | 4.0884e-05 |
| system_31 | RK4 (TP10) | 2.940 | 1.3932e-09 | 10.000 | 1.3356e-05 |
| system_31 | RK4 (TP11) | 4.390 | 1.5137e-10 | 10.000 | 1.457e-06 |
| system_31 | RK4 (TP12) | 6.870 | 1.5606e-11 | 10.000 | 1.5259e-07 |
| system_31 | RK4 (TP13) | 10.920 | 1.6551e-12 | 10.000 | 1.6821e-08 |
| system_31 | RK4 (TP14) | 16.650 | 3.2484e-13 | 10.000 | 3.6533e-09 |
| system_31 | LA3 (TP8) | 1.250 | 7.1058e-08 | 10.000 | 8.3149e-05 |
| system_31 | LA3 (TP9) | 1.780 | 7.6279e-09 | 10.000 | 4.4857e-05 |
| system_31 | LA3 (TP10) | 2.670 | 8.5419e-10 | 10.000 | 1.2882e-05 |
| system_31 | LA3 (TP11) | 4.270 | 9.3902e-11 | 10.000 | 1.3788e-06 |
| system_31 | LA3 (TP12) | 6.580 | 9.7906e-12 | 10.000 | 1.4299e-07 |
| system_31 | LA3 (TP13) | 10.140 | 1.0789e-12 | 10.000 | 1.5696e-08 |
| system_31 | LA3 (TP14) | 16.130 | 2.7699e-13 | 10.000 | 3.3895e-09 |
| system_31 | LC3 (TP8) | 1.090 | 1.7432e-07 | 10.000 | 4.6058e-05 |
| system_31 | LC3 (TP9) | 1.490 | 1.9061e-08 | 10.000 | 4.6763e-05 |
| system_31 | LC3 (TP10) | 2.220 | 1.9137e-09 | 10.000 | 1.9889e-05 |
| system_31 | LC3 (TP11) | 3.450 | 1.8493e-10 | 10.000 | 1.9633e-06 |
| system_31 | LC3 (TP12) | 5.250 | 1.7977e-11 | 10.000 | 1.9146e-07 |
| system_31 | LC3 (TP13) | 8.310 | 1.821e-12 | 10.000 | 1.9509e-08 |
| system_31 | LC3 (TP14) | 12.850 | 3.1761e-13 | 10.000 | 3.486e-09 |
| system_31 | Riot (05, 1e-11) | 0m8.552s | 1.3195e-10 | -0.000 | 3.7849e-08 |
| system_31 | Riot (10, 1e-11) | 0m4.423s | 4.2645e-12 | -0.000 | 5.8043e-09 |
| system_31 | Riot (15, 1e-11) | 0m4.983s | 1.8874e-15 | -0.000 | 1.2535e-10 |
| system_31 | Valencia-IVP (0.00025) | 0m55.912s | 0.0020183 | 4.793 | 1.5566 |
| system_31 | Valencia-IVP (0.0025) | 0m4.192s | 0.020632 | 3.252 | 1.8903 |
| system_31 | Valencia-IVP (0.025) | 0m0.399s | 0.25275 | 1.800 | 1.0445 |
| system_31 | VNODE-LP (15, 1e-14,1e-14) | 0m0.160s | 9.26e-15 | 10.000 | 1.3792e-13 |
| system_31 | VNODE-LP (20, 1e-14,1e-14) | 0m0.181s | 4.9093e-15 | 10.000 | 9.2898e-14 |
| system_31 | VNODE-LP (25, 1e-14,1e-14) | 0m0.205s | 4.0697e-15 | 10.000 | 7.63e-14 |

Table 3.28: Simulation results of Problem 32

| Problems | Methods | c5t | c5w | c6t | c6w |
|-----------|----------------------------|-----------|------------|--------|------------|
| system_32 | TAYLOR4 (TP8) | 0.110 | 1.0131e-07 | 10.000 | 1.092e-06 |
| system_32 | TAYLOR4 (TP9) | 0.150 | 1.1876e-08 | 10.000 | 1.2968e-07 |
| system_32 | TAYLOR4 (TP10) | 0.220 | 1.5521e-09 | 10.000 | 1.3318e-08 |
| system_32 | TAYLOR4 (TP11) | 0.330 | 1.5477e-10 | 10.000 | 1.4345e-09 |
| system_32 | TAYLOR4 (TP12) | 0.520 | 1.9744e-11 | 10.000 | 1.544e-10 |
| system_32 | TAYLOR4 (TP13) | 0.900 | 2.9185e-12 | 10.000 | 1.8198e-11 |
| system_32 | TAYLOR4 (TP14) | 1.280 | 1.549e-12 | 10.000 | 4.4076e-12 |
| system_32 | RK4 (TP8) | 0.050 | 1.1569e-07 | 10.000 | 1.6157e-06 |
| system_32 | RK4 (TP9) | 0.070 | 1.2497e-08 | 10.000 | 2.1179e-07 |
| system_32 | RK4 (TP10) | 0.110 | 1.4826e-09 | 10.000 | 2.7584e-08 |
| system_32 | RK4 (TP11) | 0.170 | 1.5983e-10 | 10.000 | 3.5086e-09 |
| system_32 | RK4 (TP12) | 0.250 | 1.6631e-11 | 10.000 | 4.1913e-10 |
| system_32 | RK4 (TP13) | 0.400 | 1.9895e-12 | 10.000 | 5.1035e-11 |
| system_32 | RK4 (TP14) | 0.630 | 4.9205e-13 | 10.000 | 6.4764e-12 |
| system_32 | LA3 (TP8) | 0.060 | 4.8721e-08 | 10.000 | 9.5298e-07 |
| system_32 | LA3 (TP9) | 0.080 | 5.2775e-09 | 10.000 | 9.6145e-08 |
| system_32 | LA3 (TP10) | 0.130 | 5.5251e-10 | 10.000 | 1.0333e-08 |
| system_32 | LA3 (TP11) | 0.180 | 5.6823e-11 | 10.000 | 1.1945e-09 |
| system_32 | LA3 (TP12) | 0.290 | 5.9162e-12 | 10.000 | 1.2948e-10 |
| system_32 | LA3 (TP13) | 0.460 | 8.0025e-13 | 10.000 | 1.4912e-11 |
| system_32 | LA3 (TP14) | 0.710 | 4.0501e-13 | 10.000 | 2.1458e-12 |
| system_32 | LC3 (TP8) | 0.060 | 8.8322e-08 | 10.000 | 1.0067e-06 |
| system_32 | LC3 (TP9) | 0.080 | 1.0516e-08 | 10.000 | 1.0107e-07 |
| system_32 | LC3 (TP10) | 0.110 | 1.3559e-09 | 10.000 | 1.2095e-08 |
| system_32 | LC3 (TP11) | 0.180 | 1.5019e-10 | 10.000 | 1.3537e-09 |
| system_32 | LC3 (TP12) | 0.250 | 1.6706e-11 | 10.000 | 1.5879e-10 |
| system_32 | LC3 (TP13) | 0.390 | 2.0792e-12 | 10.000 | 1.7959e-11 |
| system_32 | LC3 (TP14) | 0.610 | 4.7784e-13 | 10.000 | 2.528e-12 |
| system_32 | Riot (05, 1e-11) | 0m2.160s | 8.7466e-11 | -0.000 | 2.9713e-10 |
| system_32 | Riot (10, 1e-11) | 0m0.781s | 1.2124e-13 | -0.000 | 4.0483e-11 |
| system_32 | Riot (15, 1e-11) | 0m0.815s | 1.3411e-13 | -0.000 | 1.8493e-11 |
| system_32 | Valencia-IVP (0.00025) | 1m35.630s | 0.00026492 | 10.000 | 0.28978 |
| system_32 | Valencia-IVP (0.0025) | 0m2.151s | 0.0026499 | 10.000 | 2.9143 |
| system_32 | Valencia-IVP (0.025) | 0m0.272s | 0.026604 | 10.000 | 31.409 |
| system_32 | VNODE-LP (15, 1e-14,1e-14) | 0m0.039s | 9.77e-15 | 10.000 | 6.3727e-14 |
| system_32 | VNODE-LP (20, 1e-14,1e-14) | 0m0.044s | 8.8818e-15 | 10.000 | 7.3386e-14 |
| system_32 | VNODE-LP (25, 1e-14,1e-14) | 0m0.040s | 7.9936e-15 | 10.000 | 3.586e-14 |

Table 3.29: Simulation results of Problem 33

| Problems | Methods | c5t | c5w | c6t | c6w |
|-----------|----------------------------|-----------|---------|--------|---------|
| system_33 | TAYLOR4 (TP8) | 0.110 | 0.81192 | 10.000 | 0.20314 |
| system_33 | TAYLOR4 (TP9) | 0.150 | 0.81192 | 10.000 | 0.20314 |
| system_33 | TAYLOR4 (TP10) | 0.230 | 0.81192 | 10.000 | 0.20314 |
| system_33 | TAYLOR4 (TP11) | 0.360 | 0.81192 | 10.000 | 0.20314 |
| system_33 | TAYLOR4 (TP12) | 0.530 | 0.81192 | 10.000 | 0.20314 |
| system_33 | TAYLOR4 (TP13) | 0.860 | 0.81192 | 10.000 | 0.20314 |
| system_33 | TAYLOR4 (TP14) | 1.330 | 0.81192 | 10.000 | 0.20314 |
| system_33 | RK4 (TP8) | 0.060 | 0.81192 | 10.000 | 0.20315 |
| system_33 | RK4 (TP9) | 0.080 | 0.81192 | 10.000 | 0.20314 |
| system_33 | RK4 (TP10) | 0.110 | 0.81192 | 10.000 | 0.20314 |
| system_33 | RK4 (TP11) | 0.170 | 0.81192 | 10.000 | 0.20314 |
| system_33 | RK4 (TP12) | 0.270 | 0.81192 | 10.000 | 0.20314 |
| system_33 | RK4 (TP13) | 0.410 | 0.81192 | 10.000 | 0.20314 |
| system_33 | RK4 (TP14) | 0.640 | 0.81192 | 10.000 | 0.20314 |
| system_33 | LA3 (TP8) | 0.060 | 0.81202 | 10.000 | 0.20448 |
| system_33 | LA3 (TP9) | 0.080 | 0.81197 | 10.000 | 0.20373 |
| system_33 | LA3 (TP10) | 0.130 | 0.81194 | 10.000 | 0.20338 |
| system_33 | LA3 (TP11) | 0.200 | 0.81193 | 10.000 | 0.20324 |
| system_33 | LA3 (TP12) | 0.300 | 0.81193 | 10.000 | 0.20318 |
| system_33 | LA3 (TP13) | 0.470 | 0.81192 | 10.000 | 0.20316 |
| system_33 | LA3 (TP14) | 0.740 | 0.81192 | 10.000 | 0.20315 |
| system_33 | LC3 (TP8) | 0.060 | 0.81211 | 10.000 | 0.20522 |
| system_33 | LC3 (TP9) | 0.080 | 0.812 | 10.000 | 0.204 |
| system_33 | LC3 (TP10) | 0.120 | 0.81196 | 10.000 | 0.20349 |
| system_33 | LC3 (TP11) | 0.170 | 0.81194 | 10.000 | 0.20329 |
| system_33 | LC3 (TP12) | 0.260 | 0.81193 | 10.000 | 0.2032 |
| system_33 | LC3 (TP13) | 0.400 | 0.81192 | 10.000 | 0.20317 |
| system_33 | LC3 (TP14) | 0.620 | 0.81192 | 10.000 | 0.20315 |
| system_33 | Riot (05, 1e-11) | 0m3.466s | 0.81192 | -0.000 | 0.20314 |
| system_33 | Riot (10, 1e-11) | 0m0.842s | 0.81192 | -0.000 | 0.20314 |
| system_33 | Riot (15, 1e-11) | 0m0.886s | 0.81192 | -0.000 | 0.20314 |
| system_33 | Valencia-IVP (0.00025) | 1m30.726s | 0.8123 | 10.000 | 243.87 |
| system_33 | Valencia-IVP (0.0025) | 0m1.521s | 0.81566 | 10.000 | 249.32 |
| system_33 | Valencia-IVP (0.025) | 0m0.257s | 0.85019 | 10.000 | 309.55 |
| system_33 | VNODE-LP (15, 1e-14,1e-14) | 0m0.041s | 0.81192 | 10.000 | 0.20314 |
| system_33 | VNODE-LP (20, 1e-14,1e-14) | 0m0.042s | 0.81192 | 10.000 | 0.20314 |
| system_33 | VNODE-LP (25, 1e-14,1e-14) | 0m0.039s | 0.81192 | 10.000 | 0.20314 |

Table 3.30: Simulation results of Problem 34

| Problems | Methods | c5t | c5w | c6t | c6w |
|-----------|----------------------------|-----------|------------|--------|------------|
| system_34 | TAYLOR4 (TP8) | 0.010 | 4.7235e-09 | 10.000 | 2.9591e-07 |
| system_34 | TAYLOR4 (TP9) | 0.010 | 7.8377e-10 | 10.000 | 3.0655e-08 |
| system_34 | TAYLOR4 (TP10) | 0.020 | 1.0829e-10 | 10.000 | 3.5486e-09 |
| system_34 | TAYLOR4 (TP11) | 0.020 | 1.2753e-11 | 10.000 | 4.2103e-10 |
| system_34 | TAYLOR4 (TP12) | 0.030 | 1.3936e-12 | 10.000 | 4.4459e-11 |
| system_34 | TAYLOR4 (TP13) | 0.040 | 2.1538e-13 | 10.000 | 7.7094e-12 |
| system_34 | TAYLOR4 (TP14) | 0.070 | 1.2879e-13 | 10.000 | 5.361e-12 |
| system_34 | RK4 (TP8) | 0.010 | 1.4271e-09 | 10.000 | 7.7418e-08 |
| system_34 | RK4 (TP9) | 0.010 | 2.0589e-10 | 10.000 | 8.1568e-09 |
| system_34 | RK4 (TP10) | 0.020 | 2.1419e-11 | 10.000 | 8.3585e-10 |
| system_34 | RK4 (TP11) | 0.020 | 2.3852e-12 | 10.000 | 8.5283e-11 |
| system_34 | RK4 (TP12) | 0.030 | 2.78e-13 | 10.000 | 9.3454e-12 |
| system_34 | RK4 (TP13) | 0.040 | 6.2172e-14 | 10.000 | 1.9824e-12 |
| system_34 | RK4 (TP14) | 0.060 | 5.5955e-14 | 10.000 | 1.8456e-12 |
| system_34 | LA3 (TP8) | 0.010 | 8.073e-11 | 10.000 | 1.7684e-07 |
| system_34 | LA3 (TP9) | 0.010 | 6.0024e-11 | 10.000 | 2.3486e-08 |
| system_34 | LA3 (TP10) | 0.010 | 5.8509e-12 | 10.000 | 2.36e-09 |
| system_34 | LA3 (TP11) | 0.020 | 5.4801e-13 | 10.000 | 2.6067e-10 |
| system_34 | LA3 (TP12) | 0.020 | 7.7716e-14 | 10.000 | 2.9845e-11 |
| system_34 | LA3 (TP13) | 0.030 | 3.0642e-14 | 10.000 | 4.0146e-12 |
| system_34 | LA3 (TP14) | 0.050 | 3.6859e-14 | 10.000 | 1.5241e-12 |
| system_34 | LC3 (TP8) | 0.010 | 4.5581e-10 | 10.000 | 1.7673e-07 |
| system_34 | LC3 (TP9) | 0.010 | 1.0584e-10 | 10.000 | 1.8314e-08 |
| system_34 | LC3 (TP10) | 0.020 | 1.5158e-11 | 10.000 | 2.4613e-09 |
| system_34 | LC3 (TP11) | 0.020 | 1.7217e-12 | 10.000 | 2.8973e-10 |
| system_34 | LC3 (TP12) | 0.010 | 1.9762e-13 | 10.000 | 2.8706e-11 |
| system_34 | LC3 (TP13) | 0.040 | 4.7074e-14 | 10.000 | 4.4125e-12 |
| system_34 | LC3 (TP14) | 0.060 | 4.4409e-14 | 10.000 | 1.6751e-12 |
| system_34 | Riot (05, 1e-11) | 0m0.304s | 1.3289e-12 | -0.000 | 1.8114e-10 |
| system_34 | Riot (10, 1e-11) | 0m0.241s | 5.7954e-14 | -0.000 | 3.439e-12 |
| system_34 | Riot (15, 1e-11) | 0m0.268s | 6.9944e-14 | -0.000 | 6.0574e-13 |
| system_34 | Valencia-IVP (0.00025) | 0m42.641s | 1.6439e-05 | 10.000 | 0.0004796 |
| system_34 | Valencia-IVP (0.0025) | 0m1.277s | 0.00016439 | 10.000 | 0.0047963 |
| system_34 | Valencia-IVP (0.025) | 0m0.165s | 0.001644 | 10.000 | 0.047992 |
| system_34 | VNODE-LP (15, 1e-14,1e-14) | 0m0.008s | 8.8818e-16 | 10.000 | 3.5527e-14 |
| system_34 | VNODE-LP (20, 1e-14,1e-14) | 0m0.010s | 8.8818e-16 | 10.000 | 3.6415e-14 |
| system_34 | VNODE-LP (25, 1e-14,1e-14) | 0m0.009s | 8.8818e-16 | 10.000 | 2.931e-14 |

Table 3.31: Simulation results of Problem 35

| Problems | Methods | c5t | c5w | c6t | c6w |
|-----------|----------------------------|-----------|---------|--------|--------|
| system_35 | TAYLOR4 (TP8) | 0.010 | 0.94449 | 10.000 | 4.7953 |
| system_35 | TAYLOR4 (TP9) | 0.020 | 0.94449 | 10.000 | 4.795 |
| system_35 | TAYLOR4 (TP10) | 0.030 | 0.94449 | 10.000 | 4.7949 |
| system_35 | TAYLOR4 (TP11) | 0.030 | 0.94449 | 10.000 | 4.7949 |
| system_35 | TAYLOR4 (TP12) | 0.050 | 0.94449 | 10.000 | 4.7948 |
| system_35 | TAYLOR4 (TP13) | 0.070 | 0.94449 | 10.000 | 4.7948 |
| system_35 | TAYLOR4 (TP14) | 0.090 | 0.94449 | 10.000 | 4.7948 |
| system_35 | RK4 (TP8) | 0.010 | 0.94449 | 10.000 | 4.7948 |
| system_35 | RK4 (TP9) | 0.020 | 0.94449 | 10.000 | 4.7948 |
| system_35 | RK4 (TP10) | 0.020 | 0.94449 | 10.000 | 4.7948 |
| system_35 | RK4 (TP11) | 0.020 | 0.94449 | 10.000 | 4.7948 |
| system_35 | RK4 (TP12) | 0.040 | 0.94449 | 10.000 | 4.7948 |
| system_35 | RK4 (TP13) | 0.050 | 0.94449 | 10.000 | 4.7948 |
| system_35 | RK4 (TP14) | 0.070 | 0.94449 | 10.000 | 4.7948 |
| system_35 | LA3 (TP8) | 0.010 | 0.94461 | 10.000 | 4.8179 |
| system_35 | LA3 (TP9) | 0.010 | 0.9446 | 10.000 | 4.809 |
| system_35 | LA3 (TP10) | 0.010 | 0.94456 | 10.000 | 4.8035 |
| system_35 | LA3 (TP11) | 0.020 | 0.94454 | 10.000 | 4.8002 |
| system_35 | LA3 (TP12) | 0.030 | 0.94452 | 10.000 | 4.7982 |
| system_35 | LA3 (TP13) | 0.040 | 0.94451 | 10.000 | 4.7969 |
| system_35 | LA3 (TP14) | 0.050 | 0.9445 | 10.000 | 4.7962 |
| system_35 | LC3 (TP8) | 0.010 | 0.94473 | 10.000 | 4.8375 |
| system_35 | LC3 (TP9) | 0.010 | 0.94466 | 10.000 | 4.8213 |
| system_35 | LC3 (TP10) | 0.020 | 0.94461 | 10.000 | 4.8111 |
| system_35 | LC3 (TP11) | 0.020 | 0.94457 | 10.000 | 4.805 |
| system_35 | LC3 (TP12) | 0.030 | 0.94454 | 10.000 | 4.8012 |
| system_35 | LC3 (TP13) | 0.030 | 0.94452 | 10.000 | 4.7988 |
| system_35 | LC3 (TP14) | 0.060 | 0.94451 | 10.000 | 4.7973 |
| system_35 | Riot (05, 1e-11) | 0m26.070s | 0.93958 | -0.000 | 4.3033 |
| system_35 | Riot (10, 1e-11) | 0m21.763s | 0.93958 | -0.000 | 4.3033 |
| system_35 | Riot (15, 1e-11) | 0m1.415s | 0.93958 | -0.000 | 4.3033 |
| system_35 | Valencia-IVP (0.00025) | 0m46.038s | 0.93957 | 10.000 | 4.2038 |
| system_35 | Valencia-IVP (0.0025) | 0m1.842s | 0.93976 | 10.000 | 4.2101 |
| system_35 | Valencia-IVP (0.025) | 0m0.161s | 0.94163 | 10.000 | 4.2741 |
| system_35 | VNODE-LP (15, 1e-14,1e-14) | 0m0.010s | 0.94965 | 10.000 | 5.8441 |
| system_35 | VNODE-LP (20, 1e-14,1e-14) | 0m0.008s | 0.94965 | 10.000 | 5.8753 |
| system_35 | VNODE-LP (25, 1e-14,1e-14) | 0m0.011s | 0.94965 | 10.000 | 5.8753 |

Table 3.32: Simulation results of Problem 36

| Problems | Methods | c5t | c5w | c6t | c6w |
|-----------|----------------------------|-----------|------------|--------|------------|
| system_36 | TAYLOR4 (TP8) | 0.390 | 3.8302e-07 | 10.000 | 3.3818e-06 |
| system_36 | TAYLOR4 (TP9) | 0.480 | 5.0076e-08 | 10.000 | 4.2414e-07 |
| system_36 | TAYLOR4 (TP10) | 0.600 | 6.2302e-09 | 10.000 | 5.0646e-08 |
| system_36 | TAYLOR4 (TP11) | 0.860 | 7.2726e-10 | 10.000 | 5.7903e-09 |
| system_36 | TAYLOR4 (TP12) | 1.210 | 8.1487e-11 | 10.000 | 6.4198e-10 |
| system_36 | TAYLOR4 (TP13) | 1.790 | 2.7196e-11 | 10.000 | 1.2778e-10 |
| system_36 | TAYLOR4 (TP14) | 2.760 | 1.3596e-11 | 10.000 | 5.2596e-11 |
| system_36 | RK4 (TP8) | 0.300 | 2.8762e-07 | 10.000 | 2.7084e-06 |
| system_36 | RK4 (TP9) | 0.310 | 4.616e-08 | 10.000 | 3.7416e-07 |
| system_36 | RK4 (TP10) | 0.380 | 6.6357e-09 | 10.000 | 4.8865e-08 |
| system_36 | RK4 (TP11) | 0.480 | 8.5565e-10 | 10.000 | 5.9451e-09 |
| system_36 | RK4 (TP12) | 0.630 | 1.0328e-10 | 10.000 | 6.8487e-10 |
| system_36 | RK4 (TP13) | 0.880 | 1.2269e-11 | 10.000 | 7.8007e-11 |
| system_36 | RK4 (TP14) | 1.240 | 1.7035e-12 | 10.000 | 1.1177e-11 |
| system_36 | LA3 (TP8) | 0.300 | 2.2169e-07 | 10.000 | 2.0499e-06 |
| system_36 | LA3 (TP9) | 0.330 | 3.3496e-08 | 10.000 | 2.7089e-07 |
| system_36 | LA3 (TP10) | 0.390 | 4.619e-09 | 10.000 | 3.451e-08 |
| system_36 | LA3 (TP11) | 0.510 | 5.7408e-10 | 10.000 | 4.0876e-09 |
| system_36 | LA3 (TP12) | 0.650 | 6.7628e-11 | 10.000 | 4.6189e-10 |
| system_36 | LA3 (TP13) | 0.910 | 7.7591e-12 | 10.000 | 5.2355e-11 |
| system_36 | LA3 (TP14) | 1.420 | 1.231e-12 | 10.000 | 8.5425e-12 |
| system_36 | LC3 (TP8) | 0.290 | 2.2911e-07 | 10.000 | 2.0717e-06 |
| system_36 | LC3 (TP9) | 0.330 | 3.4984e-08 | 10.000 | 2.7398e-07 |
| system_36 | LC3 (TP10) | 0.390 | 4.9685e-09 | 10.000 | 3.4966e-08 |
| system_36 | LC3 (TP11) | 0.500 | 6.4688e-10 | 10.000 | 4.22e-09 |
| system_36 | LC3 (TP12) | 0.660 | 7.7153e-11 | 10.000 | 4.8028e-10 |
| system_36 | LC3 (TP13) | 0.890 | 9.0523e-12 | 10.000 | 5.493e-11 |
| system_36 | LC3 (TP14) | 1.320 | 1.3678e-12 | 10.000 | 8.8445e-12 |
| system_36 | Riot (05, 1e-11) | 0m1.095s | 3.8821e-11 | -0.000 | 2.6445e-10 |
| system_36 | Riot (10, 1e-11) | 0m0.857s | 2.176e-13 | -0.000 | 4.5475e-12 |
| system_36 | Riot (15, 1e-11) | 0m1.818s | 3.1442e-13 | -0.000 | 1.2212e-12 |
| system_36 | Valencia-IVP (0.00025) | 1m34.728s | 8.8326e-05 | 10.000 | 0.00054692 |
| system_36 | Valencia-IVP (0.0025) | 0m1.368s | 0.00088326 | 10.000 | 0.0054692 |
| system_36 | Valencia-IVP (0.025) | 0m0.178s | 0.0088326 | 10.000 | 0.054692 |
| system_36 | VNODE-LP (15, 1e-14,1e-14) | 0m0.014s | 1.3323e-14 | 10.000 | 9.4147e-14 |
| system_36 | VNODE-LP (20, 1e-14,1e-14) | 0m0.014s | 1.1546e-14 | 10.000 | 8.0824e-14 |
| system_36 | VNODE-LP (25, 1e-14,1e-14) | 0m0.014s | 7.9936e-15 | 10.000 | 5.9508e-14 |

Table 3.33: Simulation results of Problem 37

| Problems | Methods | c5t | c5w | c6t | c6w |
|-----------|----------------------------|-----------|---------|--------|---------|
| system_37 | TAYLOR4 (TP8) | 0.460 | 0.36452 | 10.000 | 1.0893 |
| system_37 | TAYLOR4 (TP9) | 0.570 | 0.3637 | 10.000 | 1.0799 |
| system_37 | TAYLOR4 (TP10) | 0.800 | 0.36315 | 10.000 | 1.0739 |
| system_37 | TAYLOR4 (TP11) | 1.140 | 0.36279 | 10.000 | 1.0699 |
| system_37 | TAYLOR4 (TP12) | 1.690 | 0.36255 | 10.000 | 1.0674 |
| system_37 | TAYLOR4 (TP13) | 2.530 | 0.3624 | 10.000 | 1.0658 |
| system_37 | TAYLOR4 (TP14) | 3.930 | 0.3623 | 10.000 | 1.0648 |
| system_37 | RK4 (TP8) | 0.290 | 0.36214 | 10.000 | 1.063 |
| system_37 | RK4 (TP9) | 0.360 | 0.36214 | 10.000 | 1.0631 |
| system_37 | RK4 (TP10) | 0.370 | 0.36214 | 10.000 | 1.0631 |
| system_37 | RK4 (TP11) | 0.560 | 0.36214 | 10.000 | 1.0631 |
| system_37 | RK4 (TP12) | 0.780 | 0.36214 | 10.000 | 1.0631 |
| system_37 | RK4 (TP13) | 1.090 | 0.36214 | 10.000 | 1.0631 |
| system_37 | RK4 (TP14) | 1.630 | 0.36214 | 10.000 | 1.0631 |
| system_37 | LA3 (TP8) | 0.320 | 0.36125 | 10.000 | 1.0578 |
| system_37 | LA3 (TP9) | 0.300 | 0.36152 | 10.000 | 1.0594 |
| system_37 | LA3 (TP10) | 0.430 | 0.36172 | 10.000 | 1.0606 |
| system_37 | LA3 (TP11) | 0.590 | 0.36186 | 10.000 | 1.0615 |
| system_37 | LA3 (TP12) | 0.800 | 0.36196 | 10.000 | 1.062 |
| system_37 | LA3 (TP13) | 1.150 | 0.36202 | 10.000 | 1.0624 |
| system_37 | LA3 (TP14) | 1.740 | 0.36207 | 10.000 | 1.0627 |
| system_37 | LC3 (TP8) | 0.310 | 0.36119 | 10.000 | 1.0583 |
| system_37 | LC3 (TP9) | 0.360 | 0.36148 | 10.000 | 1.0598 |
| system_37 | LC3 (TP10) | 0.450 | 0.36169 | 10.000 | 1.0609 |
| system_37 | LC3 (TP11) | 0.590 | 0.36184 | 10.000 | 1.0616 |
| system_37 | LC3 (TP12) | 0.780 | 0.36195 | 10.000 | 1.0621 |
| system_37 | LC3 (TP13) | 1.200 | 0.36201 | 10.000 | 1.0625 |
| system_37 | LC3 (TP14) | 1.710 | 0.36206 | 10.000 | 1.0627 |
| system_37 | Riot (05, 1e-11) | 1m11.410s | 0.25904 | -0.000 | 0.51435 |
| system_37 | Riot (10, 1e-11) | 0m5.525s | 0.25904 | -0.000 | 0.51435 |
| system_37 | Riot (15, 1e-11) | 0m20.456s | 0.25904 | -0.000 | 0.51435 |
| system_37 | Valencia-IVP (0.00025) | 1m26.397s | 0.25956 | 10.000 | 0.51575 |
| system_37 | Valencia-IVP (0.0025) | 0m1.774s | 0.26021 | 10.000 | 0.52027 |
| system_37 | Valencia-IVP (0.025) | 0m0.170s | 0.26796 | 10.000 | 0.56814 |
| system_37 | VNODE-LP (15, 1e-14,1e-14) | 0m0.012s | 0.26197 | 10.000 | 0.53714 |
| system_37 | VNODE-LP (20, 1e-14,1e-14) | 0m0.014s | 0.26206 | 10.000 | 0.53773 |
| system_37 | VNODE-LP (25, 1e-14,1e-14) | 0m0.015s | 0.26225 | 10.000 | 0.53846 |

Table 3.34: Simulation results of Problem 38

| Problems | Methods | c5t | c5w | c6t | c6w |
|-----------|----------------------------|-----------|------------|--------|------------|
| system_38 | TAYLOR4 (TP8) | 0.170 | 2.7922e-08 | 10.000 | 7.4431e-08 |
| system_38 | TAYLOR4 (TP9) | 0.220 | 3.2672e-09 | 10.000 | 7.942e-09 |
| system_38 | TAYLOR4 (TP10) | 0.270 | 4.1428e-10 | 10.000 | 8.3748e-10 |
| system_38 | TAYLOR4 (TP11) | 0.400 | 4.6597e-11 | 10.000 | 8.5737e-11 |
| system_38 | TAYLOR4 (TP12) | 0.560 | 5.4421e-12 | 10.000 | 8.7429e-12 |
| system_38 | TAYLOR4 (TP13) | 0.870 | 6.8723e-13 | 10.000 | 9.297e-13 |
| system_38 | TAYLOR4 (TP14) | 1.320 | 2.669e-13 | 10.000 | 1.6706e-13 |
| system_38 | RK4 (TP8) | 0.120 | 2.1332e-08 | 10.000 | 4.498e-08 |
| system_38 | RK4 (TP9) | 0.160 | 2.19e-09 | 10.000 | 4.8124e-09 |
| system_38 | RK4 (TP10) | 0.220 | 2.2353e-10 | 10.000 | 4.9613e-10 |
| system_38 | RK4 (TP11) | 0.300 | 2.2414e-11 | 10.000 | 5.1359e-11 |
| system_38 | RK4 (TP12) | 0.430 | 2.2782e-12 | 10.000 | 5.2231e-12 |
| system_38 | RK4 (TP13) | 0.620 | 2.7422e-13 | 10.000 | 5.4531e-13 |
| system_38 | RK4 (TP14) | 0.930 | 9.8588e-14 | 10.000 | 8.3211e-14 |
| system_38 | LA3 (TP8) | 0.090 | 2.3226e-08 | 10.000 | 5.5003e-08 |
| system_38 | LA3 (TP9) | 0.120 | 2.0602e-09 | 10.000 | 5.7806e-09 |
| system_38 | LA3 (TP10) | 0.160 | 1.8549e-10 | 10.000 | 5.8628e-10 |
| system_38 | LA3 (TP11) | 0.220 | 1.7422e-11 | 10.000 | 6.0634e-11 |
| system_38 | LA3 (TP12) | 0.280 | 1.6807e-12 | 10.000 | 6.1279e-12 |
| system_38 | LA3 (TP13) | 0.420 | 1.9051e-13 | 10.000 | 6.3527e-13 |
| system_38 | LA3 (TP14) | 0.600 | 6.2839e-14 | 10.000 | 8.3517e-14 |
| system_38 | LC3 (TP8) | 0.100 | 3.1915e-08 | 10.000 | 3.6288e-08 |
| system_38 | LC3 (TP9) | 0.080 | 2.9665e-09 | 10.000 | 3.5504e-09 |
| system_38 | LC3 (TP10) | 0.180 | 3.4742e-10 | 10.000 | 3.6358e-10 |
| system_38 | LC3 (TP11) | 0.240 | 3.4815e-11 | 10.000 | 3.6803e-11 |
| system_38 | LC3 (TP12) | 0.310 | 3.511e-12 | 10.000 | 3.7114e-12 |
| system_38 | LC3 (TP13) | 0.480 | 3.8325e-13 | 10.000 | 3.8405e-13 |
| system_38 | LC3 (TP14) | 0.690 | 8.8596e-14 | 10.000 | 6.0923e-14 |
| system_38 | Riot (05, 1e-11) | 0m1.119s | 8.3338e-11 | -0.000 | 3.9802e-10 |
| system_38 | Riot (10, 1e-11) | 0m0.599s | 3.0975e-14 | -0.000 | 2.307e-11 |
| system_38 | Riot (15, 1e-11) | 0m0.755s | 4.4409e-15 | -0.000 | 4.7198e-14 |
| system_38 | Valencia-IVP (0.00025) | 1m10.629s | 0.00053855 | 9.927 | 935.08 |
| system_38 | Valencia-IVP (0.0025) | 0m4.512s | 0.0054036 | 7.390 | 83.458 |
| system_38 | Valencia-IVP (0.025) | 0m0.436s | 0.055881 | 4.675 | 9.5271 |
| system_38 | VNODE-LP (15, 1e-14,1e-14) | 0m0.027s | 2.3315e-15 | 10.000 | 1.7986e-14 |
| system_38 | VNODE-LP (20, 1e-14,1e-14) | 0m0.023s | 1.4433e-15 | 10.000 | 1.2323e-14 |
| system_38 | VNODE-LP (25, 1e-14,1e-14) | 0m0.026s | 1.4155e-15 | 10.000 | 1.1435e-14 |

Table 3.35: Simulation results of Problem 39

| Problems | Methods | c5t | c5w | c6t | c6w |
|-----------|----------------------------|-----------|----------|--------|-----------|
| system_39 | TAYLOR4 (TP8) | 0.180 | 0.098956 | 10.000 | 0.0040241 |
| system_39 | TAYLOR4 (TP9) | 0.250 | 0.098918 | 10.000 | 0.0040059 |
| system_39 | TAYLOR4 (TP10) | 0.350 | 0.098895 | 10.000 | 0.0039945 |
| system_39 | TAYLOR4 (TP11) | 0.500 | 0.098881 | 10.000 | 0.0039875 |
| system_39 | TAYLOR4 (TP12) | 0.760 | 0.098872 | 10.000 | 0.0039831 |
| system_39 | TAYLOR4 (TP13) | 1.150 | 0.098866 | 10.000 | 0.0039803 |
| system_39 | TAYLOR4 (TP14) | 1.730 | 0.098863 | 10.000 | 0.0039785 |
| system_39 | RK4 (TP8) | 0.110 | 0.098857 | 10.000 | 0.0039757 |
| system_39 | RK4 (TP9) | 0.170 | 0.098857 | 10.000 | 0.0039756 |
| system_39 | RK4 (TP10) | 0.230 | 0.098857 | 10.000 | 0.0039756 |
| system_39 | RK4 (TP11) | 0.310 | 0.098856 | 10.000 | 0.0039756 |
| system_39 | RK4 (TP12) | 0.480 | 0.098856 | 10.000 | 0.0039756 |
| system_39 | RK4 (TP13) | 0.690 | 0.098856 | 10.000 | 0.0039756 |
| system_39 | RK4 (TP14) | 1.060 | 0.098856 | 10.000 | 0.0039756 |
| system_39 | LA3 (TP8) | 0.100 | 0.10458 | 10.000 | 0.0045642 |
| system_39 | LA3 (TP9) | 0.130 | 0.10263 | 10.000 | 0.0043234 |
| system_39 | LA3 (TP10) | 0.180 | 0.10129 | 10.000 | 0.0041864 |
| system_39 | LA3 (TP11) | 0.240 | 0.1004 | 10.000 | 0.0041045 |
| system_39 | LA3 (TP12) | 0.330 | 0.099845 | 10.000 | 0.0040554 |
| system_39 | LA3 (TP13) | 0.470 | 0.099481 | 10.000 | 0.0040253 |
| system_39 | LA3 (TP14) | 0.690 | 0.09925 | 10.000 | 0.0040067 |
| system_39 | LC3 (TP8) | 0.110 | 0.10484 | 10.000 | 0.0046338 |
| system_39 | LC3 (TP9) | 0.150 | 0.10273 | 10.000 | 0.0043531 |
| system_39 | LC3 (TP10) | 0.200 | 0.10128 | 10.000 | 0.0042001 |
| system_39 | LC3 (TP11) | 0.270 | 0.1004 | 10.000 | 0.004112 |
| system_39 | LC3 (TP12) | 0.380 | 0.099832 | 10.000 | 0.0040597 |
| system_39 | LC3 (TP13) | 0.560 | 0.099472 | 10.000 | 0.0040278 |
| system_39 | LC3 (TP14) | 0.820 | 0.099244 | 10.000 | 0.0040082 |
| system_39 | Riot (05, 1e-11) | 0m3.777s | 0.09197 | -0.000 | 1.135e-05 |
| system_39 | Riot (10, 1e-11) | 6m32.012s | 0.09682 | -0.000 | 0.24626 |
| system_39 | Riot (15, 1e-11) | 13m4.722s | 0.09682 | -0.000 | 0.24626 |
| system_39 | Valencia-IVP (0.00025) | 0m23.487s | 0.67999 | 2.515 | 881.5 |
| system_39 | Valencia-IVP (0.0025) | 0m1.379s | 0.68374 | 2.303 | 6.9672 |
| system_39 | Valencia-IVP (0.025) | 0m0.247s | 0.73359 | 2.275 | 9.8884 |
| system_39 | VNODE-LP (15, 1e-14,1e-14) | 0m0.028s | 0.10211 | 10.000 | 0.29379 |
| system_39 | VNODE-LP (20, 1e-14,1e-14) | 0m0.028s | 0.10278 | 10.000 | 0.30109 |
| system_39 | VNODE-LP (25, 1e-14,1e-14) | 0m0.025s | 0.10322 | 10.000 | 0.3087 |

Table 3.36: Simulation results of Problem 40

| Problems | Methods | c5t | c5w | c6t | c6w |
|-----------|----------------------------|------------|------------|--------|------------|
| system_40 | TAYLOR4 (TP8) | 25.590 | 1.5611e-06 | 9.522 | 0.62151 |
| system_40 | TAYLOR4 (TP9) | 35.300 | 1.886e-07 | 10.000 | 0.24996 |
| system_40 | TAYLOR4 (TP10) | 49.740 | 2.2374e-08 | 10.000 | 0.029197 |
| system_40 | TAYLOR4 (TP11) | 74.140 | 2.6015e-09 | 10.000 | 0.0094394 |
| system_40 | TAYLOR4 (TP12) | 113.610 | 2.9517e-10 | 10.000 | 0.0011596 |
| system_40 | TAYLOR4 (TP13) | 173.010 | 3.3894e-11 | 10.000 | 0.0001425 |
| system_40 | TAYLOR4 (TP14) | 266.180 | 5.7838e-12 | 10.000 | 2.3248e-05 |
| system_40 | RK4 (TP8) | 18.480 | 1.0075e-06 | 10.000 | 0.54239 |
| system_40 | RK4 (TP9) | 21.920 | 1.3601e-07 | 10.000 | 0.1267 |
| system_40 | RK4 (TP10) | 30.550 | 1.5332e-08 | 10.000 | 0.023281 |
| system_40 | RK4 (TP11) | 43.010 | 1.6852e-09 | 10.000 | 0.0039165 |
| system_40 | RK4 (TP12) | 64.720 | 1.8093e-10 | 10.000 | 0.00066257 |
| system_40 | RK4 (TP13) | 101.010 | 1.9397e-11 | 10.000 | 7.4275e-05 |
| system_40 | RK4 (TP14) | 151.690 | 2.5571e-12 | 10.000 | 9.829e-06 |
| system_40 | LA3 (TP8) | 16.790 | 1.1266e-06 | 10.000 | 0.60409 |
| system_40 | LA3 (TP9) | 21.760 | 1.3529e-07 | 10.000 | 0.15636 |
| system_40 | LA3 (TP10) | 29.580 | 1.6075e-08 | 10.000 | 0.026108 |
| system_40 | LA3 (TP11) | 41.390 | 1.8938e-09 | 10.000 | 0.0063105 |
| system_40 | LA3 (TP12) | 60.530 | 2.1847e-10 | 10.000 | 0.00083967 |
| system_40 | LA3 (TP13) | 91.840 | 2.4841e-11 | 10.000 | 0.00010056 |
| system_40 | LA3 (TP14) | 136.740 | 3.209e-12 | 10.000 | 1.2899e-05 |
| system_40 | LC3 (TP8) | 16.910 | 1.2305e-06 | 9.878 | 0.61908 |
| system_40 | LC3 (TP9) | 21.020 | 1.5037e-07 | 10.000 | 0.16173 |
| system_40 | LC3 (TP10) | 28.780 | 1.8244e-08 | 10.000 | 0.033471 |
| system_40 | LC3 (TP11) | 39.800 | 2.1564e-09 | 10.000 | 0.0072601 |
| system_40 | LC3 (TP12) | 58.290 | 2.5061e-10 | 10.000 | 0.0009416 |
| system_40 | LC3 (TP13) | 88.440 | 2.8377e-11 | 10.000 | 0.00011561 |
| system_40 | LC3 (TP14) | 133.470 | 3.5767e-12 | 10.000 | 1.4508e-05 |
| system_40 | Riot (05, 1e-11) | 0m26.087s | 1.9465e-10 | 0.000 | 0 |
| system_40 | Riot (10, 1e-11) | 11m50.212s | 5.0149e-12 | 0.000 | 0 |
| system_40 | Riot (15, 1e-11) | 60m12.975s | 7.1054e-15 | 0.000 | 0 |
| system_40 | Valencia-IVP (0.00025) | 0m12.132s | 0.0010009 | 0.000 | 0 |
| system_40 | Valencia-IVP (0.0025) | 0m0.366s | 0.010036 | 0.000 | 0 |
| system_40 | Valencia-IVP (0.025) | 0m0.035s | 0.10322 | 0.000 | 0 |
| system_40 | VNODE-LP (15, 1e-14,1e-14) | 0m0.046s | 3.8192e-14 | 3.000 | 2.9702e-10 |
| system_40 | VNODE-LP (20, 1e-14,1e-14) | 0m3.850s | 2.7978e-14 | 0.000 | 0 |
| system_40 | VNODE-LP (25, 1e-14,1e-14) | 0m4.400s | 2.1316e-14 | 0.000 | 0 |

Table 3.37: Simulation results of Problem 41

| Problems | Methods | c5t | c5w | c6t | c6w |
|-----------|----------------------------|-------------|---------|-------|---------|
| system.41 | TAYLOR4 (TP8) | 38.240 | 0.23341 | 2.193 | 0.81586 |
| system.41 | TAYLOR4 (TP9) | 56.240 | 0.23356 | 2.144 | 0.84571 |
| system.41 | TAYLOR4 (TP10) | 82.870 | 0.23312 | 2.150 | 0.844 |
| system.41 | TAYLOR4 (TP11) | 129.130 | 0.23292 | 2.133 | 0.86159 |
| system.41 | TAYLOR4 (TP12) | 203.650 | 0.23286 | 2.139 | 0.85592 |
| system.41 | TAYLOR4 (TP13) | 317.250 | 0.2329 | 2.131 | 0.85896 |
| system.41 | TAYLOR4 (TP14) | 501.260 | 0.23288 | 2.155 | 0.84068 |
| system.41 | RK4 (TP8) | 24.250 | 0.23001 | 2.229 | 0.79366 |
| system.41 | RK4 (TP9) | 28.080 | 0.22991 | 2.223 | 0.79461 |
| system.41 | RK4 (TP10) | 40.330 | 0.22957 | 2.211 | 0.80802 |
| system.41 | RK4 (TP11) | 59.630 | 0.23089 | 2.190 | 0.8191 |
| system.41 | RK4 (TP12) | 90.700 | 0.23209 | 2.165 | 0.83117 |
| system.41 | RK4 (TP13) | 144.140 | 0.23228 | 2.156 | 0.83803 |
| system.41 | RK4 (TP14) | 224.750 | 0.23232 | 2.149 | 0.84697 |
| system.41 | LA3 (TP8) | 22.090 | 0.23611 | 2.198 | 0.81753 |
| system.41 | LA3 (TP9) | 27.740 | 0.23533 | 2.193 | 0.81736 |
| system.41 | LA3 (TP10) | 40.040 | 0.23467 | 2.157 | 0.83501 |
| system.41 | LA3 (TP11) | 59.630 | 0.23386 | 2.143 | 0.84664 |
| system.41 | LA3 (TP12) | 90.130 | 0.23336 | 2.160 | 0.83456 |
| system.41 | LA3 (TP13) | 139.080 | 0.23337 | 2.149 | 0.84272 |
| system.41 | LA3 (TP14) | 223.950 | 0.2331 | 2.150 | 0.84385 |
| system.41 | LC3 (TP8) | 20.800 | 0.23786 | 2.181 | 0.821 |
| system.41 | LC3 (TP9) | 29.050 | 0.23605 | 2.187 | 0.8203 |
| system.41 | LC3 (TP10) | 40.420 | 0.23551 | 2.158 | 0.83568 |
| system.41 | LC3 (TP11) | 59.040 | 0.23434 | 2.140 | 0.84825 |
| system.41 | LC3 (TP12) | 91.580 | 0.23364 | 2.156 | 0.84324 |
| system.41 | LC3 (TP13) | 142.150 | 0.23351 | 2.141 | 0.85052 |
| system.41 | LC3 (TP14) | 220.420 | 0.23324 | 2.140 | 0.85526 |
| system.41 | Riot (05, 1e-11) | 4m0.951s | 0.22004 | 0.000 | 0 |
| system.41 | Riot (10, 1e-11) | 81m51.368s | 0.22004 | 0.000 | 0 |
| system.41 | Riot (15, 1e-11) | 305m35.205s | 0.22004 | 0.000 | 0 |
| system.41 | Valencia-IVP (0.00025) | 0m10.623s | 0.3966 | 0.000 | 0 |
| system.41 | Valencia-IVP (0.0025) | 0m0.275s | 0.4067 | 0.000 | 0 |
| system.41 | Valencia-IVP (0.025) | 0m0.029s | 0.51161 | 0.000 | 0 |
| system.41 | VNODE-LP (15, 1e-14,1e-14) | 0m0.056s | 0.24701 | 2.251 | 1.0915 |
| system.41 | VNODE-LP (20, 1e-14,1e-14) | 0m0.061s | 0.24758 | 2.240 | 1.1135 |
| system.41 | VNODE-LP (25, 1e-14,1e-14) | 0m0.068s | 0.24797 | 2.231 | 1.1282 |

Table 3.38: Simulation results of Problem 42

| Problems | Methods | c5t | c5w | c6t | c6w |
|-----------|----------------------------|----------|------------|--------|------------|
| system_42 | TAYLOR4 (TP8) | 0.670 | 6.5182e-08 | 10.000 | 0.0001853 |
| system_42 | TAYLOR4 (TP9) | 0.970 | 6.9772e-09 | 10.000 | 7.5231e-05 |
| system_42 | TAYLOR4 (TP10) | 1.510 | 7.267e-10 | 10.000 | 2.3754e-05 |
| system_42 | TAYLOR4 (TP11) | 2.340 | 7.4017e-11 | 10.000 | 3.93e-06 |
| system_42 | TAYLOR4 (TP12) | 3.720 | 7.7236e-12 | 10.000 | 4.2675e-07 |
| system_42 | TAYLOR4 (TP13) | 5.810 | 1.1897e-12 | 10.000 | 6.9992e-08 |
| system_42 | TAYLOR4 (TP14) | 9.150 | 7.8271e-13 | 10.000 | 4.9014e-08 |
| system_42 | RK4 (TP8) | 0.420 | 4.9849e-08 | 10.000 | 6.6443e-05 |
| system_42 | RK4 (TP9) | 0.600 | 5.3644e-09 | 10.000 | 4.1734e-05 |
| system_42 | RK4 (TP10) | 0.880 | 5.6878e-10 | 10.000 | 2.3717e-05 |
| system_42 | RK4 (TP11) | 1.360 | 5.7543e-11 | 10.000 | 3.2772e-06 |
| system_42 | RK4 (TP12) | 2.140 | 5.9515e-12 | 10.000 | 3.4427e-07 |
| system_42 | RK4 (TP13) | 3.380 | 6.9655e-13 | 10.000 | 4.1534e-08 |
| system_42 | RK4 (TP14) | 5.250 | 2.3448e-13 | 10.000 | 1.4537e-08 |
| system_42 | LA3 (TP8) | 0.380 | 5.4075e-08 | 10.000 | 0.00017238 |
| system_42 | LA3 (TP9) | 0.530 | 5.9316e-09 | 10.000 | 4.4174e-05 |
| system_42 | LA3 (TP10) | 0.800 | 6.3172e-10 | 10.000 | 2.4504e-05 |
| system_42 | LA3 (TP11) | 1.180 | 6.4227e-11 | 10.000 | 3.3231e-06 |
| system_42 | LA3 (TP12) | 1.840 | 6.5794e-12 | 10.000 | 3.4972e-07 |
| system_42 | LA3 (TP13) | 2.940 | 7.5517e-13 | 10.000 | 4.1783e-08 |
| system_42 | LA3 (TP14) | 4.530 | 2.1938e-13 | 10.000 | 1.3757e-08 |
| system_42 | LC3 (TP8) | 0.360 | 5.3813e-08 | 10.000 | 0.00013458 |
| system_42 | LC3 (TP9) | 0.480 | 5.0378e-09 | 10.000 | 7.047e-05 |
| system_42 | LC3 (TP10) | 0.700 | 4.7183e-10 | 10.000 | 2.1268e-05 |
| system_42 | LC3 (TP11) | 1.030 | 4.4507e-11 | 10.000 | 2.4301e-06 |
| system_42 | LC3 (TP12) | 1.600 | 4.3485e-12 | 10.000 | 2.4349e-07 |
| system_42 | LC3 (TP13) | 2.530 | 4.9805e-13 | 10.000 | 2.9361e-08 |
| system_42 | LC3 (TP14) | 3.990 | 1.7741e-13 | 10.000 | 1.1287e-08 |
| system_42 | Riot (05, 1e-11) | 0m0.410s | 1.4272e-10 | -0.000 | 2.2876e-08 |
| system_42 | Riot (10, 1e-11) | 0m0.197s | 4.0634e-14 | -0.000 | 1.0613e-09 |
| system_42 | Riot (15, 1e-11) | 0m0.264s | 1.8874e-15 | -0.000 | 1.1936e-09 |
| system_42 | Valencia-IVP (0.00025) | 0m4.192s | 0.00030347 | 9.119 | 981.67 |
| system_42 | Valencia-IVP (0.0025) | 0m0.741s | 0.0030419 | 7.175 | 270.69 |
| system_42 | Valencia-IVP (0.025) | 0m0.118s | 0.031193 | 5.000 | 19.406 |
| system_42 | VNODE-LP (15, 1e-14,1e-14) | 0m0.010s | 5.5511e-15 | 10.000 | 3.5123e-12 |
| system_42 | VNODE-LP (20, 1e-14,1e-14) | 0m0.007s | 3.7748e-15 | 10.000 | 2.3554e-12 |
| system_42 | VNODE-LP (25, 1e-14,1e-14) | 0m0.010s | 3.6637e-15 | 10.000 | 2.6627e-12 |

Table 3.39: Simulation results of Problem 43

| Problems | Methods | c5t | c5w | c6t | c6w |
|-----------|----------------------------|-------------|---------|-------|--------|
| system_43 | TAYLOR4 (TP8) | 0.950 | 0.40394 | 3.329 | 6930.6 |
| system_43 | TAYLOR4 (TP9) | 1.460 | 0.40494 | 3.308 | 4411.4 |
| system_43 | TAYLOR4 (TP10) | 2.430 | 0.40469 | 3.306 | 2799.3 |
| system_43 | TAYLOR4 (TP11) | 3.590 | 0.4051 | 3.299 | 1762.2 |
| system_43 | TAYLOR4 (TP12) | 5.760 | 0.4055 | 3.289 | 1112.5 |
| system_43 | TAYLOR4 (TP13) | 9.260 | 0.40555 | 3.287 | 701.1 |
| system_43 | TAYLOR4 (TP14) | 14.500 | 0.40561 | 3.280 | 442.54 |
| system_43 | RK4 (TP8) | 0.530 | 0.40361 | 3.338 | 8726.8 |
| system_43 | RK4 (TP9) | 0.800 | 0.40434 | 3.333 | 5561.3 |
| system_43 | RK4 (TP10) | 1.200 | 0.40498 | 3.316 | 3507.6 |
| system_43 | RK4 (TP11) | 1.850 | 0.40521 | 3.301 | 2218.8 |
| system_43 | RK4 (TP12) | 2.930 | 0.40515 | 3.288 | 1405.5 |
| system_43 | RK4 (TP13) | 4.610 | 0.4055 | 3.282 | 888.14 |
| system_43 | RK4 (TP14) | 7.290 | 0.40557 | 3.279 | 560.43 |
| system_43 | LA3 (TP8) | 0.490 | 0.41849 | 3.263 | 11235 |
| system_43 | LA3 (TP9) | 0.680 | 0.41437 | 3.282 | 7043.4 |
| system_43 | LA3 (TP10) | 1.030 | 0.41131 | 3.280 | 4481.6 |
| system_43 | LA3 (TP11) | 1.590 | 0.40936 | 3.277 | 2831.3 |
| system_43 | LA3 (TP12) | 2.470 | 0.40803 | 3.276 | 1786.1 |
| system_43 | LA3 (TP13) | 3.930 | 0.40718 | 3.283 | 1129.3 |
| system_43 | LA3 (TP14) | 6.240 | 0.40664 | 3.276 | 713.89 |
| system_43 | LC3 (TP8) | 0.460 | 0.42315 | 3.230 | 10495 |
| system_43 | LC3 (TP9) | 0.660 | 0.41741 | 3.254 | 6704.5 |
| system_43 | LC3 (TP10) | 0.990 | 0.41374 | 3.259 | 4299.6 |
| system_43 | LC3 (TP11) | 1.500 | 0.41082 | 3.267 | 2691.3 |
| system_43 | LC3 (TP12) | 2.310 | 0.40895 | 3.271 | 1711.2 |
| system_43 | LC3 (TP13) | 3.680 | 0.40774 | 3.271 | 1079.2 |
| system_43 | LC3 (TP14) | 5.800 | 0.40699 | 3.272 | 680.82 |
| system_43 | Riot (05, 1e-11) | 0m57.400s | 0.36095 | 0.000 | 0 |
| system_43 | Riot (10, 1e-11) | 42m34.441s | 0.36736 | 0.000 | 0 |
| system_43 | Riot (15, 1e-11) | 335m18.382s | 0.36736 | 0.000 | 0 |
| system_43 | Valencia-IVP (0.00025) | 0m4.077s | 0.63512 | 2.885 | 954.65 |
| system_43 | Valencia-IVP (0.0025) | 0m0.463s | 0.63944 | 2.860 | 300.15 |
| system_43 | Valencia-IVP (0.025) | 0m0.116s | 0.68415 | 2.650 | 29.66 |
| system_43 | VNODE-LP (15, 1e-14,1e-14) | 0m0.010s | 0.55406 | 2.715 | 13888 |
| system_43 | VNODE-LP (20, 1e-14,1e-14) | 0m0.011s | 0.55889 | 2.580 | 29046 |
| system_43 | VNODE-LP (25, 1e-14,1e-14) | 0m0.009s | 0.52831 | 2.438 | 43755 |

Table 3.40: Simulation results of Problem 44

| Problems | Methods | c5t | c5w | c6t | c6w |
|-----------|----------------------------|-----------|------------|--------|------------|
| system_44 | TAYLOR4 (TP8) | 99.250 | 1.6729e-08 | 10.000 | 5.1709e-07 |
| system_44 | TAYLOR4 (TP9) | 147.970 | 1.7394e-09 | 10.000 | 7.0242e-08 |
| system_44 | TAYLOR4 (TP10) | 226.180 | 1.7919e-10 | 10.000 | 9.4333e-09 |
| system_44 | TAYLOR4 (TP11) | 350.780 | 1.8123e-11 | 10.000 | 1.2442e-09 |
| system_44 | TAYLOR4 (TP12) | 555.550 | 1.82e-12 | 10.000 | 1.6044e-10 |
| system_44 | TAYLOR4 (TP13) | 878.990 | 1.9962e-13 | 10.000 | 2.0348e-11 |
| system_44 | TAYLOR4 (TP14) | 1374.220 | 1.9529e-13 | 10.000 | 2.7485e-12 |
| system_44 | RK4 (TP8) | 87.650 | 2.171e-08 | 10.000 | 6.5343e-07 |
| system_44 | RK4 (TP9) | 130.540 | 2.2593e-09 | 10.000 | 8.9434e-08 |
| system_44 | RK4 (TP10) | 200.110 | 2.3279e-10 | 10.000 | 1.2006e-08 |
| system_44 | RK4 (TP11) | 312.950 | 2.3621e-11 | 10.000 | 1.5783e-09 |
| system_44 | RK4 (TP12) | 493.220 | 2.3793e-12 | 10.000 | 2.0472e-10 |
| system_44 | RK4 (TP13) | 783.320 | 2.4301e-13 | 10.000 | 2.5788e-11 |
| system_44 | RK4 (TP14) | 1230.930 | 7.9048e-14 | 10.000 | 3.2618e-12 |
| system_44 | LA3 (TP8) | 71.940 | 1.6981e-08 | 10.000 | 5.8303e-07 |
| system_44 | LA3 (TP9) | 98.920 | 1.7519e-09 | 10.000 | 6.9277e-08 |
| system_44 | LA3 (TP10) | 149.090 | 1.7923e-10 | 10.000 | 8.6749e-09 |
| system_44 | LA3 (TP11) | 233.660 | 1.8224e-11 | 10.000 | 1.1029e-09 |
| system_44 | LA3 (TP12) | 361.000 | 1.8307e-12 | 10.000 | 1.3978e-10 |
| system_44 | LA3 (TP13) | 9.390 | 0 | 0.000 | 0 |
| system_44 | LA3 (TP14) | 13.960 | 0 | 0.000 | 0 |
| system_44 | LC3 (TP8) | 75.260 | 1.7199e-08 | 10.000 | 6.3209e-07 |
| system_44 | LC3 (TP9) | 106.040 | 1.7313e-09 | 10.000 | 7.4196e-08 |
| system_44 | LC3 (TP10) | 163.720 | 1.801e-10 | 10.000 | 9.1825e-09 |
| system_44 | LC3 (TP11) | 248.310 | 1.8118e-11 | 10.000 | 1.157e-09 |
| system_44 | LC3 (TP12) | 9.380 | 0 | 0.000 | 0 |
| system_44 | LC3 (TP13) | 9.390 | 0 | 0.000 | 0 |
| system_44 | LC3 (TP14) | 13.940 | 0 | 0.000 | 0 |
| system_44 | Riot | | | | |
| system_44 | Valencia-IVP (0.00025) | 0m17.732s | 0.00067987 | 8.555 | 999.95 |
| system_44 | Valencia-IVP (0.0025) | 0m1.845s | 0.0068261 | 7.338 | 997.5 |
| system_44 | Valencia-IVP (0.025) | 0m0.222s | 0.071092 | 6.000 | 977.47 |
| system_44 | VNODE-LP (15, 1e-14,1e-14) | 0m0.026s | 8.3267e-16 | 10.000 | 1.0658e-14 |
| system_44 | VNODE-LP (20, 1e-14,1e-14) | 0m0.019s | 4.996e-16 | 10.000 | 5.5511e-15 |
| system_44 | VNODE-LP (25, 1e-14,1e-14) | 0m0.014s | 1.9429e-16 | 10.000 | 3.9968e-15 |

Table 3.41: Simulation results of Problem 45

| Problems | Methods | c5t | c5w | c6t | c6w |
|-----------|----------------------------|-----------|---------|--------|---------|
| system_45 | TAYLOR4 (TP8) | 109.900 | 0.36788 | 10.000 | 0.1126 |
| system_45 | TAYLOR4 (TP9) | 162.030 | 0.36788 | 10.000 | 0.1126 |
| system_45 | TAYLOR4 (TP10) | 247.300 | 0.36788 | 10.000 | 0.1126 |
| system_45 | TAYLOR4 (TP11) | 385.000 | 0.36788 | 10.000 | 0.1126 |
| system_45 | TAYLOR4 (TP12) | 610.920 | 0.36788 | 10.000 | 0.1126 |
| system_45 | TAYLOR4 (TP13) | 978.780 | 0.36788 | 10.000 | 0.1126 |
| system_45 | TAYLOR4 (TP14) | 1538.830 | 0.36788 | 10.000 | 0.1126 |
| system_45 | RK4 (TP8) | 95.190 | 0.36788 | 10.000 | 0.1126 |
| system_45 | RK4 (TP9) | 141.530 | 0.36788 | 10.000 | 0.1126 |
| system_45 | RK4 (TP10) | 218.660 | 0.36788 | 10.000 | 0.1126 |
| system_45 | RK4 (TP11) | 338.220 | 0.36788 | 10.000 | 0.1126 |
| system_45 | RK4 (TP12) | 538.980 | 0.36788 | 10.000 | 0.1126 |
| system_45 | RK4 (TP13) | 853.390 | 0.36788 | 10.000 | 0.1126 |
| system_45 | RK4 (TP14) | 1335.820 | 0.36788 | 10.000 | 0.1126 |
| system_45 | LA3 (TP8) | 75.870 | 0.38756 | 10.000 | 0.18747 |
| system_45 | LA3 (TP9) | 106.810 | 0.38077 | 10.000 | 0.15599 |
| system_45 | LA3 (TP10) | 164.780 | 0.37601 | 10.000 | 0.13858 |
| system_45 | LA3 (TP11) | 253.600 | 0.37304 | 10.000 | 0.12843 |
| system_45 | LA3 (TP12) | 392.330 | 0.37115 | 10.000 | 0.12237 |
| system_45 | LA3 (TP13) | 628.270 | 0.36994 | 10.000 | 0.11868 |
| system_45 | LA3 (TP14) | 990.970 | 0.36917 | 10.000 | 0.1164 |
| system_45 | LC3 (TP8) | 79.310 | 0.38884 | 10.000 | 0.192 |
| system_45 | LC3 (TP9) | 114.090 | 0.38141 | 10.000 | 0.15824 |
| system_45 | LC3 (TP10) | 175.810 | 0.37636 | 10.000 | 0.13975 |
| system_45 | LC3 (TP11) | 272.780 | 0.37326 | 10.000 | 0.1291 |
| system_45 | LC3 (TP12) | 431.510 | 0.37127 | 10.000 | 0.12276 |
| system_45 | LC3 (TP13) | 676.980 | 0.37001 | 10.000 | 0.11892 |
| system_45 | LC3 (TP14) | 1073.260 | 0.36923 | 10.000 | 0.11655 |
| system_45 | Riot | | | | |
| system_45 | Valencia-IVP (0.00025) | 0m17.383s | 2.72 | 4.274 | 999.57 |
| system_45 | Valencia-IVP (0.0025) | 0m1.838s | 2.7353 | 4.263 | 997.63 |
| system_45 | Valencia-IVP (0.025) | 0m0.222s | 2.8947 | 4.150 | 973.41 |
| system_45 | VNODE-LP (15, 1e-14,1e-14) | 0m0.024s | 0.36788 | 10.000 | 0.66718 |
| system_45 | VNODE-LP (20, 1e-14,1e-14) | 0m0.020s | 0.36788 | 10.000 | 0.66718 |
| system_45 | VNODE-LP (25, 1e-14,1e-14) | 0m0.013s | 0.36788 | 10.000 | 0.66718 |

Table 3.42: Simulation results of Problem 46

| Problems | Methods | c5t | c5w | c6t | c6w |
|-----------|----------------------------|-----------|------------|--------|------------|
| system_46 | TAYLOR4 (TP8) | 206.640 | 2.8129e-07 | 10.000 | 4.6794e-08 |
| system_46 | TAYLOR4 (TP9) | 295.550 | 3.9758e-08 | 10.000 | 6.1744e-09 |
| system_46 | TAYLOR4 (TP10) | 442.500 | 5.6043e-09 | 10.000 | 9.4693e-10 |
| system_46 | TAYLOR4 (TP11) | 659.730 | 7.9915e-10 | 10.000 | 1.3713e-10 |
| system_46 | TAYLOR4 (TP12) | 1006.260 | 1.1174e-10 | 10.000 | 2.0165e-11 |
| system_46 | TAYLOR4 (TP13) | 1520.690 | 1.557e-11 | 10.000 | 2.9375e-12 |
| system_46 | TAYLOR4 (TP14) | 2311.850 | 2.162e-12 | 10.000 | 4.3671e-13 |
| system_46 | RK4 (TP8) | 181.660 | 3.5581e-07 | 10.000 | 6.0115e-08 |
| system_46 | RK4 (TP9) | 266.450 | 5.0501e-08 | 10.000 | 8.4781e-09 |
| system_46 | RK4 (TP10) | 398.290 | 7.1016e-09 | 10.000 | 1.1435e-09 |
| system_46 | RK4 (TP11) | 595.700 | 9.957e-10 | 10.000 | 1.7022e-10 |
| system_46 | RK4 (TP12) | 900.870 | 1.3983e-10 | 10.000 | 2.492e-11 |
| system_46 | RK4 (TP13) | 1372.050 | 1.9572e-11 | 10.000 | 3.7196e-12 |
| system_46 | RK4 (TP14) | 2083.170 | 2.7271e-12 | 10.000 | 5.4328e-13 |
| system_46 | LA3 (TP8) | 142.750 | 3.029e-07 | 10.000 | 8.59e-08 |
| system_46 | LA3 (TP9) | 204.050 | 3.6926e-08 | 10.000 | 7.3781e-09 |
| system_46 | LA3 (TP10) | 301.230 | 4.8977e-09 | 10.000 | 9.0584e-10 |
| system_46 | LA3 (TP11) | 455.320 | 6.5218e-10 | 10.000 | 1.2048e-10 |
| system_46 | LA3 (TP12) | 683.080 | 9.0125e-11 | 10.000 | 1.6853e-11 |
| system_46 | LA3 (TP13) | 1035.690 | 1.2352e-11 | 10.000 | 2.3525e-12 |
| system_46 | LA3 (TP14) | 18.270 | 0 | 0.000 | 0 |
| system_46 | LC3 (TP8) | 149.890 | 3.2956e-07 | 10.000 | 1.0752e-07 |
| system_46 | LC3 (TP9) | 219.390 | 4.0029e-08 | 10.000 | 8.8952e-09 |
| system_46 | LC3 (TP10) | 323.850 | 5.1723e-09 | 10.000 | 1.0649e-09 |
| system_46 | LC3 (TP11) | 490.370 | 6.9923e-10 | 10.000 | 1.2867e-10 |
| system_46 | LC3 (TP12) | 737.050 | 9.5419e-11 | 10.000 | 1.7671e-11 |
| system_46 | LC3 (TP13) | 1110.750 | 1.3104e-11 | 10.000 | 2.5332e-12 |
| system_46 | LC3 (TP14) | 18.420 | 0 | 0.000 | 0 |
| system_46 | Riot | | | | |
| system_46 | Valencia-IVP (0.00025) | 0m19.620s | 0.90083 | 1.613 | 998.27 |
| system_46 | Valencia-IVP (0.0025) | 0m2.097s | 10.696 | 1.383 | 994.33 |
| system_46 | Valencia-IVP (0.025) | 0m0.280s | 717.1 | 1.000 | 717.1 |
| system_46 | VNODE-LP (15, 1e-14,1e-14) | 0m0.112s | 2.9109e-15 | 10.000 | 8.7708e-14 |
| system_46 | VNODE-LP (20, 1e-14,1e-14) | 0m0.064s | 1.5613e-15 | 10.000 | 3.9968e-14 |
| system_46 | VNODE-LP (25, 1e-14,1e-14) | 0m0.040s | 8.3267e-16 | 10.000 | 2.4092e-14 |

Table 3.43: Simulation results of Problem 47

| Problems | Methods | c5t | c5w | c6t | c6w |
|-----------|----------------------------|-----------|----------|--------|------------|
| system_47 | TAYLOR4 (TP8) | 251.600 | 0.073576 | 10.000 | 9.138e-06 |
| system_47 | TAYLOR4 (TP9) | 370.930 | 0.073576 | 10.000 | 9.0857e-06 |
| system_47 | TAYLOR4 (TP10) | 574.250 | 0.073576 | 10.000 | 9.08e-06 |
| system_47 | TAYLOR4 (TP11) | 896.240 | 0.073576 | 10.000 | 9.08e-06 |
| system_47 | TAYLOR4 (TP12) | 1411.760 | 0.073576 | 10.000 | 9.08e-06 |
| system_47 | TAYLOR4 (TP13) | 2235.520 | 0.073576 | 10.000 | 9.08e-06 |
| system_47 | TAYLOR4 (TP14) | 3574.470 | 0.073576 | 10.000 | 9.08e-06 |
| system_47 | RK4 (TP8) | 214.830 | 0.073576 | 10.000 | 9.1474e-06 |
| system_47 | RK4 (TP9) | 329.930 | 0.073576 | 10.000 | 9.0882e-06 |
| system_47 | RK4 (TP10) | 514.490 | 0.073576 | 10.000 | 9.08e-06 |
| system_47 | RK4 (TP11) | 804.210 | 0.073576 | 10.000 | 9.08e-06 |
| system_47 | RK4 (TP12) | 1261.520 | 0.073576 | 10.000 | 9.08e-06 |
| system_47 | RK4 (TP13) | 1985.340 | 0.073576 | 10.000 | 9.08e-06 |
| system_47 | RK4 (TP14) | 3126.690 | 0.073576 | 10.000 | 9.08e-06 |
| system_47 | LA3 (TP8) | 172.260 | 0.073587 | 10.000 | 5.1859e-05 |
| system_47 | LA3 (TP9) | 253.280 | 0.073581 | 10.000 | 2.9554e-05 |
| system_47 | LA3 (TP10) | 387.100 | 0.073578 | 10.000 | 1.9645e-05 |
| system_47 | LA3 (TP11) | 606.460 | 0.073577 | 10.000 | 1.4898e-05 |
| system_47 | LA3 (TP12) | 939.230 | 0.073576 | 10.000 | 1.2458e-05 |
| system_47 | LA3 (TP13) | 1476.800 | 0.073576 | 10.000 | 1.1107e-05 |
| system_47 | LA3 (TP14) | 2318.910 | 0.073576 | 10.000 | 1.0316e-05 |
| system_47 | LC3 (TP8) | 183.610 | 0.073588 | 10.000 | 5.7105e-05 |
| system_47 | LC3 (TP9) | 275.560 | 0.073581 | 10.000 | 3.2142e-05 |
| system_47 | LC3 (TP10) | 417.670 | 0.073578 | 10.000 | 2.056e-05 |
| system_47 | LC3 (TP11) | 655.720 | 0.073577 | 10.000 | 1.5314e-05 |
| system_47 | LC3 (TP12) | 1024.290 | 0.073576 | 10.000 | 1.2655e-05 |
| system_47 | LC3 (TP13) | 1599.140 | 0.073576 | 10.000 | 1.1206e-05 |
| system_47 | LC3 (TP14) | 2513.770 | 0.073576 | 10.000 | 1.0371e-05 |
| system_47 | Riot | | | | |
| system_47 | Valencia-IVP (0.00025) | 0m19.696s | 43.149 | 1.244 | 998.7 |
| system_47 | Valencia-IVP (0.0025) | 0m2.122s | 62.436 | 1.215 | 989.09 |
| system_47 | Valencia-IVP (0.025) | 0m0.270s | 832.17 | 0.975 | 832.17 |
| system_47 | VNODE-LP (15, 1e-14,1e-14) | 0m0.112s | 0.073576 | 10.000 | 0.19992 |
| system_47 | VNODE-LP (20, 1e-14,1e-14) | 0m0.063s | 0.073576 | 10.000 | 0.19992 |
| system_47 | VNODE-LP (25, 1e-14,1e-14) | 0m0.038s | 0.073576 | 10.000 | 0.19992 |

Table 3.44: Simulation results of Problem 48

| Problems | Methods | c5t | c5w | c6t | c6w |
|-----------|----------------------------|-----------|------------|--------|------------|
| system_48 | TAYLOR4 (TP8) | 270.510 | 3.5098e-08 | 10.000 | 7.1884e-07 |
| system_48 | TAYLOR4 (TP9) | 411.140 | 3.7378e-09 | 10.000 | 1.0437e-07 |
| system_48 | TAYLOR4 (TP10) | 636.760 | 3.9037e-10 | 10.000 | 1.5031e-08 |
| system_48 | TAYLOR4 (TP11) | 1003.860 | 3.9284e-11 | 10.000 | 2.1706e-09 |
| system_48 | TAYLOR4 (TP12) | 1586.020 | 4.0629e-12 | 10.000 | 3.1246e-10 |
| system_48 | TAYLOR4 (TP13) | 2484.510 | 4.9355e-13 | 10.000 | 4.4848e-11 |
| system_48 | TAYLOR4 (TP14) | 3908.730 | 1.9457e-13 | 10.000 | 6.4941e-12 |
| system_48 | RK4 (TP8) | 244.580 | 4.4892e-08 | 10.000 | 9.0045e-07 |
| system_48 | RK4 (TP9) | 372.300 | 4.8038e-09 | 10.000 | 1.306e-07 |
| system_48 | RK4 (TP10) | 583.480 | 5.112e-10 | 10.000 | 1.8909e-08 |
| system_48 | RK4 (TP11) | 921.360 | 5.1871e-11 | 10.000 | 2.7231e-09 |
| system_48 | RK4 (TP12) | 1449.170 | 5.2396e-12 | 10.000 | 3.9235e-10 |
| system_48 | RK4 (TP13) | 2274.660 | 5.6177e-13 | 10.000 | 5.6309e-11 |
| system_48 | RK4 (TP14) | 3598.580 | 1.0719e-13 | 10.000 | 8.1017e-12 |
| system_48 | LA3 (TP8) | 192.150 | 3.5165e-08 | 10.000 | 5.7435e-06 |
| system_48 | LA3 (TP9) | 282.410 | 3.6459e-09 | 10.000 | 3.0749e-07 |
| system_48 | LA3 (TP10) | 432.900 | 3.8153e-10 | 10.000 | 2.5881e-08 |
| system_48 | LA3 (TP11) | 688.310 | 3.9668e-11 | 10.000 | 2.7242e-09 |
| system_48 | LA3 (TP12) | 1069.590 | 4.036e-12 | 10.000 | 3.2499e-10 |
| system_48 | LA3 (TP13) | 24.680 | 0 | 0.000 | 0 |
| system_48 | LA3 (TP14) | 32.600 | 0 | 0.000 | 0 |
| system_48 | LC3 (TP8) | 206.350 | 3.7145e-08 | 10.000 | 9.4115e-06 |
| system_48 | LC3 (TP9) | 303.850 | 3.7256e-09 | 10.000 | 3.9047e-07 |
| system_48 | LC3 (TP10) | 468.430 | 3.9032e-10 | 10.000 | 2.9785e-08 |
| system_48 | LC3 (TP11) | 738.850 | 3.9451e-11 | 10.000 | 3.0202e-09 |
| system_48 | LC3 (TP12) | 1151.230 | 4.015e-12 | 10.000 | 3.5442e-10 |
| system_48 | LC3 (TP13) | 24.670 | 0 | 0.000 | 0 |
| system_48 | LC3 (TP14) | 32.170 | 0 | 0.000 | 0 |
| system_48 | Riot | | | | |
| system_48 | Valencia-IVP (0.00025) | 0m24.122s | 0.004682 | 4.352 | 999.44 |
| system_48 | Valencia-IVP (0.0025) | 0m2.676s | 0.047669 | 3.725 | 994.41 |
| system_48 | Valencia-IVP (0.025) | 0m0.311s | 0.57528 | 2.950 | 913.46 |
| system_48 | VNODE-LP (15, 1e-14,1e-14) | 0m0.041s | 8.0491e-16 | 10.000 | 9.194e-16 |
| system_48 | VNODE-LP (20, 1e-14,1e-14) | 0m0.029s | 7.2164e-16 | 10.000 | 3.4001e-16 |
| system_48 | VNODE-LP (25, 1e-14,1e-14) | 0m0.023s | 3.0531e-16 | 10.000 | 2.498e-16 |

Table 3.45: Simulation results of Problem 49

| Problems | Methods | c5t | c5w | c6t | c6w |
|-----------|----------------------------|-----------|---------|--------|----------|
| system_49 | TAYLOR4 (TP8) | 309.470 | 0.10763 | 10.000 | 0.011897 |
| system_49 | TAYLOR4 (TP9) | 471.270 | 0.10763 | 10.000 | 0.011895 |
| system_49 | TAYLOR4 (TP10) | 733.000 | 0.10763 | 10.000 | 0.011895 |
| system_49 | TAYLOR4 (TP11) | 1166.480 | 0.10763 | 10.000 | 0.011895 |
| system_49 | TAYLOR4 (TP12) | 1859.450 | 0.10763 | 10.000 | 0.011895 |
| system_49 | TAYLOR4 (TP13) | 2949.570 | 0.10763 | 10.000 | 0.011895 |
| system_49 | TAYLOR4 (TP14) | 4695.330 | 0.10763 | 10.000 | 0.011895 |
| system_49 | RK4 (TP8) | 272.790 | 0.10763 | 10.000 | 0.011897 |
| system_49 | RK4 (TP9) | 422.230 | 0.10763 | 10.000 | 0.011895 |
| system_49 | RK4 (TP10) | 655.010 | 0.10763 | 10.000 | 0.011895 |
| system_49 | RK4 (TP11) | 1042.530 | 0.10763 | 10.000 | 0.011895 |
| system_49 | RK4 (TP12) | 1639.250 | 0.10763 | 10.000 | 0.011895 |
| system_49 | RK4 (TP13) | 2579.950 | 0.10763 | 10.000 | 0.011895 |
| system_49 | RK4 (TP14) | 4085.610 | 0.10763 | 10.000 | 0.011895 |
| system_49 | LA3 (TP8) | 212.960 | 0.11444 | 10.000 | 0.059049 |
| system_49 | LA3 (TP9) | 317.320 | 0.11195 | 10.000 | 0.033987 |
| system_49 | LA3 (TP10) | 489.220 | 0.11035 | 10.000 | 0.02343 |
| system_49 | LA3 (TP11) | 773.440 | 0.10933 | 10.000 | 0.018369 |
| system_49 | LA3 (TP12) | 1212.350 | 0.1087 | 10.000 | 0.015688 |
| system_49 | LA3 (TP13) | 1913.060 | 0.1083 | 10.000 | 0.01418 |
| system_49 | LA3 (TP14) | 3027.240 | 0.10805 | 10.000 | 0.013295 |
| system_49 | LC3 (TP8) | 227.850 | 0.11525 | 10.000 | 0.063765 |
| system_49 | LC3 (TP9) | 339.800 | 0.11229 | 10.000 | 0.035622 |
| system_49 | LC3 (TP10) | 526.690 | 0.11049 | 10.000 | 0.024098 |
| system_49 | LC3 (TP11) | 839.800 | 0.10941 | 10.000 | 0.018691 |
| system_49 | LC3 (TP12) | 1309.320 | 0.10874 | 10.000 | 0.015859 |
| system_49 | LC3 (TP13) | 2073.580 | 0.10833 | 10.000 | 0.014276 |
| system_49 | LC3 (TP14) | 3273.800 | 0.10807 | 10.000 | 0.013352 |
| system_49 | Riot | | | | |
| system_49 | Valencia-IVP (0.00025) | 0m24.032s | 5.8874 | 2.488 | 999.56 |
| system_49 | Valencia-IVP (0.0025) | 0m2.571s | 5.9852 | 2.475 | 998.38 |
| system_49 | Valencia-IVP (0.025) | 0m0.314s | 7.1174 | 2.350 | 997.96 |
| system_49 | VNODE-LP (15, 1e-14,1e-14) | 0m0.044s | 0.10763 | 10.000 | 0.011895 |
| system_49 | VNODE-LP (20, 1e-14,1e-14) | 0m0.030s | 0.10763 | 10.000 | 0.011895 |
| system_49 | VNODE-LP (25, 1e-14,1e-14) | 0m0.021s | 0.10763 | 10.000 | 0.011895 |

Table 3.46: Simulation results of Problem 56

| Problems | Methods | c5t | c5w | c6t | c6w |
|-----------|----------------------------|----------|------------|--------|------------|
| system_56 | TAYLOR4 (TP8) | 1.210 | 1.2465e-05 | 10.000 | 8.3213e-05 |
| system_56 | TAYLOR4 (TP9) | 1.470 | 4.9579e-06 | 10.000 | 4.0366e-05 |
| system_56 | TAYLOR4 (TP10) | 2.060 | 2.875e-06 | 10.000 | 3.0631e-05 |
| system_56 | TAYLOR4 (TP11) | 2.980 | 9.1925e-07 | 10.000 | 3.1768e-05 |
| system_56 | TAYLOR4 (TP12) | 4.320 | 2.9164e-07 | 10.000 | 0.00013262 |
| system_56 | TAYLOR4 (TP13) | 6.340 | 8.4087e-08 | 10.000 | 0.00010804 |
| system_56 | TAYLOR4 (TP14) | 9.610 | 1.159e-08 | 10.000 | 3.8782e-05 |
| system_56 | RK4 (TP8) | 0.590 | 2.9768e-07 | 10.000 | 1.6874e-05 |
| system_56 | RK4 (TP9) | 0.800 | 4.3149e-08 | 10.000 | 1.0541e-05 |
| system_56 | RK4 (TP10) | 1.040 | 6.4105e-09 | 10.000 | 8.784e-06 |
| system_56 | RK4 (TP11) | 1.550 | 8.693e-10 | 10.000 | 2.2891e-06 |
| system_56 | RK4 (TP12) | 2.300 | 1.1902e-10 | 10.000 | 3.4672e-07 |
| system_56 | RK4 (TP13) | 3.410 | 1.572e-11 | 10.000 | 4.8711e-08 |
| system_56 | RK4 (TP14) | 5.200 | 2.0466e-12 | 10.000 | 6.8005e-09 |
| system_56 | LA3 (TP8) | 0.590 | 2.3105e-07 | 10.000 | 2.6021e-05 |
| system_56 | LA3 (TP9) | 0.790 | 3.7592e-08 | 10.000 | 1.2067e-05 |
| system_56 | LA3 (TP10) | 1.090 | 5.8336e-09 | 10.000 | 9.2705e-06 |
| system_56 | LA3 (TP11) | 1.520 | 8.9354e-10 | 10.000 | 2.7425e-06 |
| system_56 | LA3 (TP12) | 2.210 | 1.3327e-10 | 10.000 | 4.3035e-07 |
| system_56 | LA3 (TP13) | 3.110 | 1.9496e-11 | 10.000 | 6.3061e-08 |
| system_56 | LA3 (TP14) | 4.550 | 2.8287e-12 | 10.000 | 9.2639e-09 |
| system_56 | LC3 (TP8) | 0.600 | 2.2727e-07 | 10.000 | 2.0461e-05 |
| system_56 | LC3 (TP9) | 0.780 | 3.6407e-08 | 10.000 | 1.6216e-05 |
| system_56 | LC3 (TP10) | 1.090 | 5.4528e-09 | 10.000 | 9.3004e-06 |
| system_56 | LC3 (TP11) | 1.570 | 7.8127e-10 | 10.000 | 2.411e-06 |
| system_56 | LC3 (TP12) | 2.350 | 1.062e-10 | 10.000 | 3.3882e-07 |
| system_56 | LC3 (TP13) | 3.370 | 1.3902e-11 | 10.000 | 4.5448e-08 |
| system_56 | LC3 (TP14) | 5.080 | 1.8026e-12 | 10.000 | 6.1851e-09 |
| system_56 | Riot (02, 1e-11) | 0m2.480s | 2.643e-07 | -0.000 | 0.001449 |
| system_56 | Riot (05, 1e-11) | 0m0.300s | 6.8263e-11 | -0.000 | 2.0833e-07 |
| system_56 | Riot (10, 1e-11) | 0m0.259s | 1.0353e-12 | -0.000 | 1.1906e-09 |
| system_56 | Riot (15, 1e-11) | 0m0.375s | 4.563e-14 | -0.000 | 6.2571e-12 |
| system_56 | Valencia-IVP (0.00025) | 0m1.982s | 0.00019354 | 10.000 | 4.7911 |
| system_56 | Valencia-IVP (0.0025) | 0m0.184s | 0.0019484 | 10.000 | 48.755 |
| system_56 | Valencia-IVP (0.025) | 0m0.026s | 0.020834 | 10.000 | 582.16 |
| system_56 | VNODE-LP (15, 1e-14,1e-14) | 0m0.015s | 4.6629e-15 | 10.000 | 6.9611e-14 |
| system_56 | VNODE-LP (20, 1e-14,1e-14) | 0m0.017s | 3.5527e-15 | 10.000 | 5.948e-14 |
| system_56 | VNODE-LP (25, 1e-14,1e-14) | 0m0.019s | 2.7756e-15 | 10.000 | 3.9801e-14 |

Table 3.47: Simulation results of Problem 57

| Problems | Methods | c5t | c5w | c6t | c6w |
|-----------|----------------------------|----------|-----------|--------|-----------|
| system_57 | TAYLOR4 (TP8) | 1.060 | 0.0067999 | 10.000 | 0.0034968 |
| system_57 | TAYLOR4 (TP9) | 1.420 | 0.0067971 | 10.000 | 0.0033828 |
| system_57 | TAYLOR4 (TP10) | 2.050 | 0.0067966 | 10.000 | 0.003375 |
| system_57 | TAYLOR4 (TP11) | 2.970 | 0.0067958 | 10.000 | 0.0033756 |
| system_57 | TAYLOR4 (TP12) | 4.230 | 0.0067957 | 10.000 | 0.0034279 |
| system_57 | TAYLOR4 (TP13) | 6.310 | 0.0067956 | 10.000 | 0.0034369 |
| system_57 | TAYLOR4 (TP14) | 9.570 | 0.0067956 | 10.000 | 0.0033881 |
| system_57 | RK4 (TP8) | 0.580 | 0.0067958 | 10.000 | 0.0033643 |
| system_57 | RK4 (TP9) | 0.780 | 0.0067956 | 10.000 | 0.0033621 |
| system_57 | RK4 (TP10) | 1.060 | 0.0067956 | 10.000 | 0.0033584 |
| system_57 | RK4 (TP11) | 1.530 | 0.0067956 | 10.000 | 0.0033523 |
| system_57 | RK4 (TP12) | 2.410 | 0.0067956 | 10.000 | 0.0033504 |
| system_57 | RK4 (TP13) | 3.320 | 0.0067956 | 10.000 | 0.0033501 |
| system_57 | RK4 (TP14) | 5.180 | 0.0067956 | 10.000 | 0.00335 |
| system_57 | LA3 (TP8) | 0.580 | 0.0069796 | 10.000 | 0.013207 |
| system_57 | LA3 (TP9) | 0.770 | 0.0069339 | 10.000 | 0.0083059 |
| system_57 | LA3 (TP10) | 1.060 | 0.0068985 | 10.000 | 0.0086576 |
| system_57 | LA3 (TP11) | 1.480 | 0.0068688 | 10.000 | 0.0060924 |
| system_57 | LA3 (TP12) | 2.160 | 0.0068461 | 10.000 | 0.0054237 |
| system_57 | LA3 (TP13) | 3.030 | 0.0068304 | 10.000 | 0.0045981 |
| system_57 | LA3 (TP14) | 4.490 | 0.0068194 | 10.000 | 0.004046 |
| system_57 | LC3 (TP8) | 0.590 | 0.0070117 | 10.000 | 0.014465 |
| system_57 | LC3 (TP9) | 0.770 | 0.0069595 | 10.000 | 0.0095625 |
| system_57 | LC3 (TP10) | 1.070 | 0.0069121 | 10.000 | 0.010461 |
| system_57 | LC3 (TP11) | 1.520 | 0.0068748 | 10.000 | 0.0065855 |
| system_57 | LC3 (TP12) | 2.280 | 0.0068492 | 10.000 | 0.0056602 |
| system_57 | LC3 (TP13) | 3.300 | 0.0068311 | 10.000 | 0.0047481 |
| system_57 | LC3 (TP14) | 5.050 | 0.0068188 | 10.000 | 0.004001 |
| system_57 | Riot (05, 1e-11) | 0m0.342s | 0.013481 | -0.000 | 33.434 |
| system_57 | Riot (10, 1e-11) | 0m0.308s | 0.012937 | -0.000 | 4.2549 |
| system_57 | Riot (15, 1e-11) | 0m0.517s | 0.012937 | -0.000 | 1.078 |
| system_57 | Valencia-IVP (0.00025) | 0m1.863s | 0.015962 | 10.000 | 288.91 |
| system_57 | Valencia-IVP (0.0025) | 0m0.180s | 0.017692 | 10.000 | 337.44 |
| system_57 | Valencia-IVP (0.025) | 0m0.024s | 0.035905 | 10.000 | 921.84 |
| system_57 | VNODE-LP (15, 1e-14,1e-14) | 0m0.015s | 0.0067956 | 10.000 | 0.054773 |
| system_57 | VNODE-LP (20, 1e-14,1e-14) | 0m0.018s | 0.0067956 | 10.000 | 0.054773 |
| system_57 | VNODE-LP (25, 1e-14,1e-14) | 0m0.018s | 0.0067956 | 10.000 | 0.054773 |

Table 3.48: Simulation results of Problem 58

| Problems | Methods | c5t | c5w | c6t | c6w |
|-----------|----------------------------|----------|------------|--------|------------|
| system_58 | TAYLOR4 (TP8) | 0.180 | 7.7141e-08 | 10.000 | 3.4549e-05 |
| system_58 | TAYLOR4 (TP9) | 0.260 | 8.0285e-09 | 10.000 | 2.1271e-05 |
| system_58 | TAYLOR4 (TP10) | 0.410 | 8.2963e-10 | 10.000 | 9.8007e-06 |
| system_58 | TAYLOR4 (TP11) | 0.640 | 8.5321e-11 | 10.000 | 3.2405e-06 |
| system_58 | TAYLOR4 (TP12) | 1.000 | 8.848e-12 | 10.000 | 3.459e-07 |
| system_58 | TAYLOR4 (TP13) | 1.550 | 1.6449e-12 | 10.000 | 5.4546e-08 |
| system_58 | TAYLOR4 (TP14) | 2.730 | 1.3616e-12 | 10.000 | 3.6384e-08 |
| system_58 | RK4 (TP8) | 0.130 | 1.1435e-07 | 10.000 | 3.0612e-05 |
| system_58 | RK4 (TP9) | 0.190 | 1.5557e-08 | 10.000 | 1.7471e-05 |
| system_58 | RK4 (TP10) | 0.280 | 1.8288e-09 | 10.000 | 8.2663e-06 |
| system_58 | RK4 (TP11) | 0.440 | 1.9948e-10 | 10.000 | 4.5881e-06 |
| system_58 | RK4 (TP12) | 0.680 | 2.3118e-11 | 10.000 | 5.8802e-07 |
| system_58 | RK4 (TP13) | 1.030 | 2.7485e-12 | 10.000 | 7.0181e-08 |
| system_58 | RK4 (TP14) | 1.600 | 6.4215e-13 | 10.000 | 1.6374e-08 |
| system_58 | LA3 (TP8) | 0.100 | 7.7582e-08 | 10.000 | 3.862e-05 |
| system_58 | LA3 (TP9) | 0.150 | 8.4071e-09 | 10.000 | 1.8014e-05 |
| system_58 | LA3 (TP10) | 0.230 | 1e-09 | 10.000 | 5.9569e-06 |
| system_58 | LA3 (TP11) | 0.350 | 1.1954e-10 | 10.000 | 4.7884e-06 |
| system_58 | LA3 (TP12) | 0.550 | 1.4015e-11 | 10.000 | 5.7684e-07 |
| system_58 | LA3 (TP13) | 0.840 | 1.6018e-12 | 10.000 | 6.6192e-08 |
| system_58 | LA3 (TP14) | 1.300 | 4.8228e-13 | 10.000 | 1.4357e-08 |
| system_58 | LC3 (TP8) | 0.110 | 7.6055e-08 | 10.000 | 3.2019e-05 |
| system_58 | LC3 (TP9) | 0.160 | 7.0379e-09 | 10.000 | 1.7252e-05 |
| system_58 | LC3 (TP10) | 0.240 | 7.073e-10 | 10.000 | 8.826e-06 |
| system_58 | LC3 (TP11) | 0.370 | 6.9519e-11 | 10.000 | 2.7407e-06 |
| system_58 | LC3 (TP12) | 0.570 | 7.0854e-12 | 10.000 | 2.6685e-07 |
| system_58 | LC3 (TP13) | 0.880 | 9.0639e-13 | 10.000 | 3.0817e-08 |
| system_58 | LC3 (TP14) | 1.360 | 4.0279e-13 | 10.000 | 1.0658e-08 |
| system_58 | Riot (05, 1e-11) | 0m0.386s | 6.7986e-11 | -0.000 | 1.8892e-06 |
| system_58 | Riot (10, 1e-11) | 0m0.225s | 7.1609e-13 | -0.000 | 3.3649e-08 |
| system_58 | Riot (15, 1e-11) | 0m0.310s | 2.1094e-14 | -0.000 | 7.9267e-07 |
| system_58 | Valencia-IVP (0.00025) | 0m1.907s | 0.0032029 | 4.129 | 968.02 |
| system_58 | Valencia-IVP (0.0025) | 0m0.289s | 0.032453 | 3.468 | 825.8 |
| system_58 | Valencia-IVP (0.025) | 0m0.063s | 0.36874 | 2.325 | 2.7348 |
| system_58 | VNODE-LP (15, 1e-14,1e-14) | 0m0.007s | 9.992e-15 | 10.000 | 5.2854e-13 |
| system_58 | VNODE-LP (20, 1e-14,1e-14) | 0m0.007s | 5.9952e-15 | 10.000 | 3.5797e-13 |
| system_58 | VNODE-LP (25, 1e-14,1e-14) | 0m0.008s | 5.107e-15 | 10.000 | 2.6821e-13 |

Table 3.49: Simulation results of Problem 59

| Problems | Methods | c5t | c5w | c6t | c6w |
|-----------|----------------------------|------------|---------|-------|------------|
| system_59 | TAYLOR4 (TP8) | 0.320 | 0.61888 | 2.332 | 409.62 |
| system_59 | TAYLOR4 (TP9) | 0.500 | 0.61763 | 2.342 | 627.45 |
| system_59 | TAYLOR4 (TP10) | 0.780 | 0.62016 | 2.343 | 947.95 |
| system_59 | TAYLOR4 (TP11) | 1.220 | 0.61634 | 2.353 | 1393.3 |
| system_59 | TAYLOR4 (TP12) | 1.960 | 0.61604 | 2.356 | 2026.9 |
| system_59 | TAYLOR4 (TP13) | 3.070 | 0.61584 | 2.358 | 2938.4 |
| system_59 | TAYLOR4 (TP14) | 4.880 | 0.61601 | 2.356 | 1820.2 |
| system_59 | RK4 (TP8) | 0.190 | 0.61554 | 2.340 | 309.16 |
| system_59 | RK4 (TP9) | 0.290 | 0.61552 | 2.346 | 482.14 |
| system_59 | RK4 (TP10) | 0.460 | 0.61552 | 2.351 | 729.25 |
| system_59 | RK4 (TP11) | 0.700 | 0.61571 | 2.354 | 1074.3 |
| system_59 | RK4 (TP12) | 1.150 | 0.61887 | 2.351 | 1577.8 |
| system_59 | RK4 (TP13) | 1.740 | 0.61551 | 2.358 | 2300 |
| system_59 | RK4 (TP14) | 2.740 | 0.61551 | 2.359 | 2656.2 |
| system_59 | LA3 (TP8) | 0.170 | 0.62868 | 2.301 | 256.97 |
| system_59 | LA3 (TP9) | 0.240 | 0.6239 | 2.322 | 405.24 |
| system_59 | LA3 (TP10) | 0.370 | 0.62081 | 2.335 | 610.27 |
| system_59 | LA3 (TP11) | 0.580 | 0.61885 | 2.344 | 914.53 |
| system_59 | LA3 (TP12) | 0.890 | 0.61796 | 2.350 | 1348.2 |
| system_59 | LA3 (TP13) | 1.410 | 0.61684 | 2.354 | 1965 |
| system_59 | LA3 (TP14) | 2.230 | 0.61635 | 2.357 | 2848.1 |
| system_59 | LC3 (TP8) | 0.170 | 0.63191 | 2.285 | 262.91 |
| system_59 | LC3 (TP9) | 0.260 | 0.6259 | 2.312 | 421.05 |
| system_59 | LC3 (TP10) | 0.390 | 0.62194 | 2.329 | 642.8 |
| system_59 | LC3 (TP11) | 0.610 | 0.61954 | 2.341 | 952.94 |
| system_59 | LC3 (TP12) | 1.050 | 0.62126 | 2.343 | 1402.2 |
| system_59 | LC3 (TP13) | 1.490 | 0.61753 | 2.352 | 2043.1 |
| system_59 | LC3 (TP14) | 2.350 | 0.61652 | 2.356 | 2965.4 |
| system_59 | Riot (05, 1e-11) | 0m7.354s | 0 | 0.000 | 0 |
| system_59 | Riot (10, 1e-11) | 6m33.869s | 0.58244 | 0.000 | 0 |
| system_59 | Riot (15, 1e-11) | 53m34.326s | 0.58244 | 0.000 | 0 |
| system_59 | Valencia-IVP (0.00025) | 0m3.563s | 1.4356 | 1.733 | 990.17 |
| system_59 | Valencia-IVP (0.0025) | 0m0.469s | 1.5086 | 1.698 | 818.5 |
| system_59 | Valencia-IVP (0.025) | 0m0.100s | 3.003 | 1.300 | 23.135 |
| system_59 | VNODE-LP (15, 1e-14,1e-14) | 0m0.013s | 1.4378 | 1.641 | 3.7929e+05 |
| system_59 | VNODE-LP (20, 1e-14,1e-14) | 0m0.013s | 1.8859 | 1.527 | 1.176e+06 |
| system_59 | VNODE-LP (25, 1e-14,1e-14) | 0m0.011s | 2.2062 | 1.455 | 1.9992e+06 |

Table 3.50: Simulation results of Problem 60

| Problems | Methods | c5t | c5w | c6t | c6w |
|-----------|----------------------------|----------|------------|--------|------------|
| system_60 | TAYLOR4 (TP8) | 0.980 | 3.6313e-08 | 10.000 | 2.0172e-05 |
| system_60 | TAYLOR4 (TP9) | 1.510 | 3.91e-09 | 10.000 | 1.0452e-05 |
| system_60 | TAYLOR4 (TP10) | 2.370 | 4.1545e-10 | 10.000 | 2.449e-06 |
| system_60 | TAYLOR4 (TP11) | 3.760 | 4.2142e-11 | 10.000 | 2.5501e-07 |
| system_60 | TAYLOR4 (TP12) | 5.900 | 4.5062e-12 | 10.000 | 2.7482e-08 |
| system_60 | TAYLOR4 (TP13) | 9.510 | 8.6153e-13 | 10.000 | 4.9282e-09 |
| system_60 | TAYLOR4 (TP14) | 15.000 | 7.2609e-13 | 10.000 | 3.8885e-09 |
| system_60 | RK4 (TP8) | 0.560 | 5.1365e-08 | 10.000 | 1.0619e-05 |
| system_60 | RK4 (TP9) | 0.850 | 5.3363e-09 | 10.000 | 6.8093e-06 |
| system_60 | RK4 (TP10) | 1.280 | 5.4613e-10 | 10.000 | 3.8676e-06 |
| system_60 | RK4 (TP11) | 2.460 | 5.5102e-11 | 10.000 | 4.6275e-07 |
| system_60 | RK4 (TP12) | 3.240 | 5.5893e-12 | 10.000 | 4.885e-08 |
| system_60 | RK4 (TP13) | 5.110 | 6.1329e-13 | 10.000 | 5.556e-09 |
| system_60 | RK4 (TP14) | 7.970 | 2.0783e-13 | 10.000 | 1.3849e-09 |
| system_60 | LA3 (TP8) | 0.550 | 5.2924e-08 | 10.000 | 1.3089e-05 |
| system_60 | LA3 (TP9) | 0.840 | 5.5683e-09 | 10.000 | 1.2154e-05 |
| system_60 | LA3 (TP10) | 1.230 | 5.7092e-10 | 10.000 | 2.8252e-06 |
| system_60 | LA3 (TP11) | 2.120 | 5.8239e-11 | 10.000 | 3.2878e-07 |
| system_60 | LA3 (TP12) | 3.040 | 5.9095e-12 | 10.000 | 3.4149e-08 |
| system_60 | LA3 (TP13) | 4.770 | 6.4371e-13 | 10.000 | 4.0022e-09 |
| system_60 | LA3 (TP14) | 7.540 | 1.9762e-13 | 10.000 | 1.2146e-09 |
| system_60 | LC3 (TP8) | 0.580 | 3.2515e-08 | 10.000 | 1.2785e-05 |
| system_60 | LC3 (TP9) | 0.850 | 3.4269e-09 | 10.000 | 1.1738e-05 |
| system_60 | LC3 (TP10) | 1.330 | 3.5506e-10 | 10.000 | 2.5568e-06 |
| system_60 | LC3 (TP11) | 2.090 | 3.6346e-11 | 10.000 | 2.6495e-07 |
| system_60 | LC3 (TP12) | 3.310 | 3.6904e-12 | 10.000 | 2.7388e-08 |
| system_60 | LC3 (TP13) | 5.160 | 4.6496e-13 | 10.000 | 3.3249e-09 |
| system_60 | LC3 (TP14) | 8.120 | 2.0606e-13 | 10.000 | 1.1795e-09 |
| system_60 | Riot (05, 1e-11) | 0m0.401s | 1.0846e-10 | -0.000 | 4.1356e-07 |
| system_60 | Riot (10, 1e-11) | 0m0.208s | 1.3138e-12 | -0.000 | 1.4383e-08 |
| system_60 | Riot (15, 1e-11) | 0m0.293s | 2.3981e-14 | -0.000 | 1.4009e-09 |
| system_60 | Valencia-IVP (0.00025) | 0m2.208s | 0.0012113 | 10.000 | 21.282 |
| system_60 | Valencia-IVP (0.0025) | 0m0.282s | 0.012152 | 8.033 | 944.65 |
| system_60 | Valencia-IVP (0.025) | 0m0.049s | 0.12493 | 5.225 | 615.14 |
| system_60 | VNODE-LP (15, 1e-14,1e-14) | 0m0.015s | 6.3283e-15 | 10.000 | 1.8436e-12 |
| system_60 | VNODE-LP (20, 1e-14,1e-14) | 0m0.013s | 5.9952e-15 | 10.000 | 2.2619e-12 |
| system_60 | VNODE-LP (25, 1e-14,1e-14) | 0m0.013s | 3.9968e-15 | 10.000 | 1.127e-12 |

Table 3.51: Simulation results of Problem 61

| Problems | Methods | c5t | c5w | c6t | c6w |
|-----------|----------------------------|-----------|-----------|--------|------------|
| system_61 | TAYLOR4 (TP8) | 1.180 | 0.0054407 | 10.000 | 8.3228 |
| system_61 | TAYLOR4 (TP9) | 1.830 | 0.0053657 | 10.000 | 15.453 |
| system_61 | TAYLOR4 (TP10) | 2.860 | 0.0054044 | 10.000 | 14.813 |
| system_61 | TAYLOR4 (TP11) | 4.600 | 0.0054132 | 10.000 | 320.66 |
| system_61 | TAYLOR4 (TP12) | 7.210 | 0.0054145 | 10.000 | 11.811 |
| system_61 | TAYLOR4 (TP13) | 11.320 | 0.0053453 | 10.000 | 313.11 |
| system_61 | TAYLOR4 (TP14) | 18.070 | 0.0054383 | 10.000 | 725.35 |
| system_61 | RK4 (TP8) | 0.620 | 0.0052643 | 10.000 | 3.3273 |
| system_61 | RK4 (TP9) | 0.920 | 0.0052883 | 10.000 | 12.87 |
| system_61 | RK4 (TP10) | 1.420 | 0.0054387 | 10.000 | 8.7345 |
| system_61 | RK4 (TP11) | 2.330 | 0.0053817 | 10.000 | 14.179 |
| system_61 | RK4 (TP12) | 3.520 | 0.0053906 | 10.000 | 90.053 |
| system_61 | RK4 (TP13) | 5.520 | 0.0054731 | 10.000 | 40.322 |
| system_61 | RK4 (TP14) | 8.640 | 0.0054604 | 10.000 | 52.883 |
| system_61 | LA3 (TP8) | 0.600 | 0.0053293 | 10.000 | 11.103 |
| system_61 | LA3 (TP9) | 0.890 | 0.0053306 | 10.000 | 15.398 |
| system_61 | LA3 (TP10) | 1.340 | 0.0053562 | 10.000 | 10.597 |
| system_61 | LA3 (TP11) | 2.080 | 0.0054059 | 10.000 | 24.382 |
| system_61 | LA3 (TP12) | 3.320 | 0.0054158 | 10.000 | 22.094 |
| system_61 | LA3 (TP13) | 5.200 | 0.0054598 | 9.972 | 75209 |
| system_61 | LA3 (TP14) | 8.110 | 0.0054368 | 10.000 | 84.479 |
| system_61 | LC3 (TP8) | 0.620 | 0.0053219 | 10.000 | 15.264 |
| system_61 | LC3 (TP9) | 0.930 | 0.0053593 | 10.000 | 13.911 |
| system_61 | LC3 (TP10) | 1.430 | 0.005359 | 10.000 | 12.418 |
| system_61 | LC3 (TP11) | 2.270 | 0.0054463 | 10.000 | 63.773 |
| system_61 | LC3 (TP12) | 3.630 | 0.0054206 | 10.000 | 26.739 |
| system_61 | LC3 (TP13) | 5.640 | 0.0054502 | 9.731 | 81583 |
| system_61 | LC3 (TP14) | 8.820 | 0.0054423 | 10.000 | 44.156 |
| system_61 | Riot (05, 1e-11) | 0m29.113s | 0.016523 | 0.000 | 0 |
| system_61 | Riot (10, 1e-11) | 2m2.447s | 0.016523 | 0.000 | 0 |
| system_61 | Riot (15, 1e-11) | 9m16.121s | 0.016523 | 0.000 | 0 |
| system_61 | Valencia-IVP (0.00025) | 0m2.193s | 0.0070886 | 7.850 | 995.84 |
| system_61 | Valencia-IVP (0.0025) | 0m0.314s | 0.018078 | 7.098 | 938.56 |
| system_61 | Valencia-IVP (0.025) | 0m0.049s | 0.13117 | 5.150 | 535.8 |
| system_61 | VNODE-LP (15, 1e-14,1e-14) | 0m0.015s | 0.0064256 | 9.464 | 1.0425e+08 |
| system_61 | VNODE-LP (20, 1e-14,1e-14) | 0m0.011s | 0.007766 | 9.213 | 4.7889e+08 |
| system_61 | VNODE-LP (25, 1e-14,1e-14) | 0m0.012s | 0.0087521 | 9.173 | 1.0624e+09 |

Table 3.52: Simulation results of Problem 62

| Problems | Methods | c5t | c5w | c6t | c6w |
|-----------|----------------------------|----------|------------|--------|------------|
| system_62 | TAYLOR4 (TP8) | 0.020 | 1.2802e-09 | 10.000 | 1.0457e-06 |
| system_62 | TAYLOR4 (TP9) | 0.020 | 1.2802e-09 | 10.000 | 1.4972e-07 |
| system_62 | TAYLOR4 (TP10) | 0.020 | 9.382e-10 | 10.000 | 1.6447e-08 |
| system_62 | TAYLOR4 (TP11) | 0.030 | 1.4039e-10 | 10.000 | 1.7013e-09 |
| system_62 | TAYLOR4 (TP12) | 0.030 | 1.5092e-11 | 10.000 | 1.7605e-10 |
| system_62 | TAYLOR4 (TP13) | 0.050 | 2.1814e-12 | 10.000 | 2.251e-11 |
| system_62 | TAYLOR4 (TP14) | 0.070 | 9.8055e-13 | 10.000 | 9.5852e-12 |
| system_62 | RK4 (TP8) | 0.020 | 7.5438e-11 | 10.000 | 6.0009e-07 |
| system_62 | RK4 (TP9) | 0.020 | 7.5438e-11 | 10.000 | 2.0981e-07 |
| system_62 | RK4 (TP10) | 0.020 | 7.5438e-11 | 10.000 | 2.6419e-08 |
| system_62 | RK4 (TP11) | 0.020 | 7.5438e-11 | 10.000 | 2.6952e-09 |
| system_62 | RK4 (TP12) | 0.020 | 1.1987e-11 | 10.000 | 2.6037e-10 |
| system_62 | RK4 (TP13) | 0.020 | 1.7266e-12 | 10.000 | 2.6858e-11 |
| system_62 | RK4 (TP14) | 0.040 | 4.0501e-13 | 10.000 | 4.7393e-12 |
| system_62 | LA3 (TP8) | 0.020 | 2.0744e-10 | 10.000 | 3.7513e-07 |
| system_62 | LA3 (TP9) | 0.020 | 2.0744e-10 | 10.000 | 6.8085e-08 |
| system_62 | LA3 (TP10) | 0.020 | 2.0744e-10 | 10.000 | 8.2075e-09 |
| system_62 | LA3 (TP11) | 0.020 | 8.3048e-11 | 10.000 | 9.0913e-10 |
| system_62 | LA3 (TP12) | 0.020 | 2.4023e-11 | 10.000 | 1.1102e-10 |
| system_62 | LA3 (TP13) | 0.030 | 3.3396e-12 | 10.000 | 1.3493e-11 |
| system_62 | LA3 (TP14) | 0.040 | 6.0396e-13 | 10.000 | 3.4959e-12 |
| system_62 | LC3 (TP8) | 0.020 | 8.3944e-11 | 10.000 | 2.7272e-07 |
| system_62 | LC3 (TP9) | 0.020 | 8.3944e-11 | 10.000 | 1.0016e-07 |
| system_62 | LC3 (TP10) | 0.020 | 8.3944e-11 | 10.000 | 1.1054e-08 |
| system_62 | LC3 (TP11) | 0.020 | 7.875e-11 | 10.000 | 1.1258e-09 |
| system_62 | LC3 (TP12) | 0.020 | 1.1283e-11 | 10.000 | 1.1067e-10 |
| system_62 | LC3 (TP13) | 0.030 | 1.5774e-12 | 10.000 | 1.2335e-11 |
| system_62 | LC3 (TP14) | 0.040 | 3.8369e-13 | 10.000 | 3.304e-12 |
| system_62 | Riot (05, 1e-11) | 0m0.096s | 7.887e-13 | -0.000 | 3.9957e-11 |
| system_62 | Riot (10, 1e-11) | 0m0.116s | 7.9226e-13 | -0.000 | 2.2027e-13 |
| system_62 | Riot (15, 1e-11) | 0m0.139s | 9.3081e-13 | -0.000 | 5.0093e-13 |
| system_62 | Valencia-IVP (0.00025) | 0m1.501s | 8e-06 | 10.000 | 9.0701e-05 |
| system_62 | Valencia-IVP (0.0025) | 0m0.135s | 8.0004e-05 | 10.000 | 0.00090724 |
| system_62 | Valencia-IVP (0.025) | 0m0.017s | 0.00080027 | 10.000 | 0.0090954 |
| system_62 | VNODE-LP (15, 1e-14,1e-14) | 0m0.006s | 1.0658e-14 | 10.000 | 1.0303e-13 |
| system_62 | VNODE-LP (20, 1e-14,1e-14) | 0m0.006s | 1.0658e-14 | 10.000 | 1.1013e-13 |
| system_62 | VNODE-LP (25, 1e-14,1e-14) | 0m0.005s | 1.0658e-14 | 10.000 | 1.1013e-13 |

Table 3.53: Simulation results of Problem 63

| Problems | Methods | c5t | c5w | c6t | c6w |
|-----------|----------------------------|----------|------------|--------|------------|
| system_63 | TAYLOR4 (TP8) | 0.050 | 0.91819 | 10.000 | 4.9513 |
| system_63 | TAYLOR4 (TP9) | 0.060 | 0.91498 | 10.000 | 4.7543 |
| system_63 | TAYLOR4 (TP10) | 0.090 | 0.9127 | 10.000 | 4.6373 |
| system_63 | TAYLOR4 (TP11) | 0.130 | 0.9111 | 10.000 | 4.5661 |
| system_63 | TAYLOR4 (TP12) | 0.200 | 0.91002 | 10.000 | 4.5221 |
| system_63 | TAYLOR4 (TP13) | 0.320 | 0.90932 | 10.000 | 4.4947 |
| system_63 | TAYLOR4 (TP14) | 0.500 | 0.90886 | 10.000 | 4.4776 |
| system_63 | RK4 (TP8) | 0.030 | 0.90814 | 10.000 | 4.452 |
| system_63 | RK4 (TP9) | 0.030 | 0.9081 | 10.000 | 4.4501 |
| system_63 | RK4 (TP10) | 0.050 | 0.90808 | 10.000 | 4.4493 |
| system_63 | RK4 (TP11) | 0.070 | 0.90808 | 10.000 | 4.449 |
| system_63 | RK4 (TP12) | 0.110 | 0.90807 | 10.000 | 4.4489 |
| system_63 | RK4 (TP13) | 0.160 | 0.90807 | 10.000 | 4.4488 |
| system_63 | RK4 (TP14) | 0.250 | 0.90807 | 10.000 | 4.4488 |
| system_63 | LA3 (TP8) | 0.030 | 0.94854 | 10.000 | 5.196 |
| system_63 | LA3 (TP9) | 0.030 | 0.93622 | 10.000 | 4.9019 |
| system_63 | LA3 (TP10) | 0.040 | 0.927 | 10.000 | 4.7292 |
| system_63 | LA3 (TP11) | 0.060 | 0.9205 | 10.000 | 4.6236 |
| system_63 | LA3 (TP12) | 0.090 | 0.91616 | 10.000 | 4.5583 |
| system_63 | LA3 (TP13) | 0.140 | 0.91325 | 10.000 | 4.5176 |
| system_63 | LA3 (TP14) | 0.220 | 0.91136 | 10.000 | 4.492 |
| system_63 | LC3 (TP8) | 0.030 | 0.95645 | 10.000 | 5.5166 |
| system_63 | LC3 (TP9) | 0.030 | 0.94129 | 10.000 | 5.0708 |
| system_63 | LC3 (TP10) | 0.050 | 0.9303 | 10.000 | 4.8246 |
| system_63 | LC3 (TP11) | 0.060 | 0.92279 | 10.000 | 4.6803 |
| system_63 | LC3 (TP12) | 0.100 | 0.91757 | 10.000 | 4.5928 |
| system_63 | LC3 (TP13) | 0.140 | 0.91414 | 10.000 | 4.5388 |
| system_63 | LC3 (TP14) | 0.230 | 0.91192 | 10.000 | 4.5052 |
| system_63 | Riot (05, 1e-11) | 0m0.226s | 6.1391e-12 | -0.000 | 2.1793e-10 |
| system_63 | Riot (10, 1e-11) | 0m0.219s | 6.1391e-12 | -0.000 | 8.3134e-13 |
| system_63 | Riot (15, 1e-11) | 0m0.222s | 3.6238e-13 | -0.000 | 3.979e-13 |
| system_63 | Valencia-IVP (0.00025) | 0m3.804s | 1.4207 | 4.983 | 939.4 |
| system_63 | Valencia-IVP (0.0025) | 0m0.416s | 1.4208 | 4.960 | 184.88 |
| system_63 | Valencia-IVP (0.025) | 0m0.067s | 1.4224 | 3.675 | 6.8657 |
| system_63 | VNODE-LP (15, 1e-14,1e-14) | 0m0.006s | 1.1898 | 5.765 | 12397 |
| system_63 | VNODE-LP (20, 1e-14,1e-14) | 0m0.006s | 1.1582 | 4.716 | 24367 |
| system_63 | VNODE-LP (25, 1e-14,1e-14) | 0m0.004s | 1.161 | 4.394 | 39403 |

Table 3.54: Simulation results of Problem 64

| Problems | Methods | c5t | c5w | c6t | c6w |
|-----------|----------------------------|----------|------------|--------|------------|
| system_64 | TAYLOR4 (TP8) | 0.390 | 1.4114e-06 | 10.000 | 0.00068375 |
| system_64 | TAYLOR4 (TP9) | 0.390 | 1.2487e-06 | 10.000 | 0.00015597 |
| system_64 | TAYLOR4 (TP10) | 0.430 | 4.3621e-07 | 10.000 | 4.6461e-05 |
| system_64 | TAYLOR4 (TP11) | 0.560 | 1.2507e-07 | 10.000 | 1.1767e-05 |
| system_64 | TAYLOR4 (TP12) | 0.730 | 3.3494e-08 | 10.000 | 2.9977e-06 |
| system_64 | TAYLOR4 (TP13) | 0.980 | 8.5987e-09 | 10.000 | 7.7019e-07 |
| system_64 | TAYLOR4 (TP14) | 1.400 | 2.3403e-09 | 10.000 | 1.9712e-07 |
| system_64 | RK4 (TP8) | 0.330 | 4.21e-11 | 10.000 | 4.7828e-07 |
| system_64 | RK4 (TP9) | 0.330 | 4.21e-11 | 10.000 | 4.7336e-08 |
| system_64 | RK4 (TP10) | 0.330 | 3.0917e-11 | 10.000 | 4.8889e-09 |
| system_64 | RK4 (TP11) | 0.400 | 4.2763e-12 | 10.000 | 5.0111e-10 |
| system_64 | RK4 (TP12) | 0.470 | 4.4072e-13 | 10.000 | 5.0163e-11 |
| system_64 | RK4 (TP13) | 0.610 | 4.842e-14 | 10.000 | 5.218e-12 |
| system_64 | RK4 (TP14) | 0.840 | 5.6413e-15 | 10.000 | 7.4518e-13 |
| system_64 | LA3 (TP8) | 0.330 | 4.4317e-11 | 10.000 | 3.2208e-07 |
| system_64 | LA3 (TP9) | 0.330 | 4.4317e-11 | 10.000 | 3.4219e-08 |
| system_64 | LA3 (TP10) | 0.360 | 2.6981e-11 | 10.000 | 3.3887e-09 |
| system_64 | LA3 (TP11) | 0.400 | 3.1899e-12 | 10.000 | 3.4486e-10 |
| system_64 | LA3 (TP12) | 0.470 | 3.5908e-13 | 10.000 | 3.4774e-11 |
| system_64 | LA3 (TP13) | 0.640 | 3.9885e-14 | 10.000 | 3.6753e-12 |
| system_64 | LA3 (TP14) | 0.890 | 4.7254e-15 | 10.000 | 6.1373e-13 |
| system_64 | LC3 (TP8) | 0.330 | 4.1986e-11 | 10.000 | 3.5873e-07 |
| system_64 | LC3 (TP9) | 0.330 | 4.1986e-11 | 10.000 | 3.7852e-08 |
| system_64 | LC3 (TP10) | 0.370 | 2.5936e-11 | 10.000 | 3.8917e-09 |
| system_64 | LC3 (TP11) | 0.400 | 3.0693e-12 | 10.000 | 3.9943e-10 |
| system_64 | LC3 (TP12) | 0.470 | 3.4297e-13 | 10.000 | 4.0659e-11 |
| system_64 | LC3 (TP13) | 0.650 | 3.849e-14 | 10.000 | 4.2375e-12 |
| system_64 | LC3 (TP14) | 0.880 | 4.5519e-15 | 10.000 | 6.6169e-13 |
| system_64 | Riot (05, 1e-11) | 0m0.136s | 3.194e-14 | -0.000 | 1.1558e-10 |
| system_64 | Riot (10, 1e-11) | 0m0.253s | 5.4123e-16 | -0.000 | 1.35e-13 |
| system_64 | Riot (15, 1e-11) | 0m0.252s | 5.4123e-16 | -0.000 | 6.9278e-14 |
| system_64 | Valencia-IVP (0.00025) | 0m1.721s | 1.0417e-05 | 10.000 | 0.00016797 |
| system_64 | Valencia-IVP (0.0025) | 0m0.165s | 0.00010417 | 10.000 | 0.0016797 |
| system_64 | Valencia-IVP (0.025) | 0m0.019s | 0.0010417 | 10.000 | 0.016797 |
| system_64 | VNODE-LP (15, 1e-14,1e-14) | 0m0.004s | 6.245e-17 | 10.000 | 9.77e-15 |
| system_64 | VNODE-LP (20, 1e-14,1e-14) | 0m0.005s | 6.9389e-17 | 10.000 | 1.199e-14 |
| system_64 | VNODE-LP (25, 1e-14,1e-14) | 0m0.004s | 6.9389e-17 | 10.000 | 1.0658e-14 |

Table 3.55: Simulation results of Problem 65

| Problems | Methods | c5t | c5w | c6t | c6w |
|-----------|----------------------------|----------|---------|--------|--------|
| system_65 | TAYLOR4 (TP8) | 0.410 | 0.25212 | 10.000 | 2.7137 |
| system_65 | TAYLOR4 (TP9) | 0.410 | 0.25212 | 10.000 | 2.7126 |
| system_65 | TAYLOR4 (TP10) | 0.500 | 0.25212 | 10.000 | 2.7121 |
| system_65 | TAYLOR4 (TP11) | 0.590 | 0.25211 | 10.000 | 2.7118 |
| system_65 | TAYLOR4 (TP12) | 0.760 | 0.25211 | 10.000 | 2.7116 |
| system_65 | TAYLOR4 (TP13) | 1.070 | 0.25211 | 10.000 | 2.7115 |
| system_65 | TAYLOR4 (TP14) | 1.570 | 0.25211 | 10.000 | 2.7114 |
| system_65 | RK4 (TP8) | 0.340 | 0.25211 | 10.000 | 2.7113 |
| system_65 | RK4 (TP9) | 0.330 | 0.25211 | 10.000 | 2.7113 |
| system_65 | RK4 (TP10) | 0.370 | 0.25211 | 10.000 | 2.7113 |
| system_65 | RK4 (TP11) | 0.400 | 0.25211 | 10.000 | 2.7113 |
| system_65 | RK4 (TP12) | 0.470 | 0.25211 | 10.000 | 2.7113 |
| system_65 | RK4 (TP13) | 0.650 | 0.25211 | 10.000 | 2.7113 |
| system_65 | RK4 (TP14) | 0.860 | 0.25211 | 10.000 | 2.7113 |
| system_65 | LA3 (TP8) | 0.330 | 0.25211 | 10.000 | 2.7134 |
| system_65 | LA3 (TP9) | 0.330 | 0.25211 | 10.000 | 2.7127 |
| system_65 | LA3 (TP10) | 0.370 | 0.25211 | 10.000 | 2.7122 |
| system_65 | LA3 (TP11) | 0.400 | 0.25211 | 10.000 | 2.7119 |
| system_65 | LA3 (TP12) | 0.500 | 0.25211 | 10.000 | 2.7117 |
| system_65 | LA3 (TP13) | 0.690 | 0.25211 | 10.000 | 2.7115 |
| system_65 | LA3 (TP14) | 0.940 | 0.25211 | 10.000 | 2.7115 |
| system_65 | LC3 (TP8) | 0.330 | 0.25211 | 10.000 | 2.7145 |
| system_65 | LC3 (TP9) | 0.330 | 0.25211 | 10.000 | 2.7133 |
| system_65 | LC3 (TP10) | 0.370 | 0.25211 | 10.000 | 2.7126 |
| system_65 | LC3 (TP11) | 0.400 | 0.25211 | 10.000 | 2.7121 |
| system_65 | LC3 (TP12) | 0.500 | 0.25211 | 10.000 | 2.7118 |
| system_65 | LC3 (TP13) | 0.650 | 0.25211 | 10.000 | 2.7117 |
| system_65 | LC3 (TP14) | 0.900 | 0.25211 | 10.000 | 2.7115 |
| system_65 | Riot (05, 1e-11) | 0m5.669s | 0.25147 | -0.000 | 2.6697 |
| system_65 | Riot (10, 1e-11) | 0m1.551s | 0.25147 | -0.000 | 2.6698 |
| system_65 | Riot (15, 1e-11) | 0m5.042s | 0.25147 | -0.000 | 2.6698 |
| system_65 | Valencia-IVP (0.00025) | 0m1.576s | 0.25147 | 10.000 | 2.6699 |
| system_65 | Valencia-IVP (0.0025) | 0m0.146s | 0.25147 | 10.000 | 2.6716 |
| system_65 | Valencia-IVP (0.025) | 0m0.021s | 0.25177 | 10.000 | 2.6883 |
| system_65 | VNODE-LP (15, 1e-14,1e-14) | 0m0.006s | 0.25278 | 10.000 | 2.7636 |
| system_65 | VNODE-LP (20, 1e-14,1e-14) | 0m0.006s | 0.25278 | 10.000 | 2.7636 |
| system_65 | VNODE-LP (25, 1e-14,1e-14) | 0m0.005s | 0.25278 | 10.000 | 2.7636 |

Table 3.56: Simulation results of Problem 71

| Problems | Methods | c5t | c5w | c6t | c6w |
|-----------|----------------------------|----------|----------|-------|----------|
| system_71 | TAYLOR4 (TP8) | 0.410 | 0.34183 | 0.723 | 0.34183 |
| system_71 | TAYLOR4 (TP9) | 0.610 | 0.34398 | 0.723 | 0.34398 |
| system_71 | TAYLOR4 (TP10) | 0.930 | 0.34513 | 0.723 | 0.34513 |
| system_71 | TAYLOR4 (TP11) | 1.530 | 0.34637 | 0.723 | 0.34637 |
| system_71 | TAYLOR4 (TP12) | 2.420 | 0.34685 | 0.723 | 0.34685 |
| system_71 | TAYLOR4 (TP13) | 3.600 | 0.34733 | 0.723 | 0.34733 |
| system_71 | TAYLOR4 (TP14) | 5.500 | 0.34747 | 0.723 | 0.34747 |
| system_71 | RK4 (TP8) | 0.410 | 0.34107 | 0.710 | 0.34107 |
| system_71 | RK4 (TP9) | 0.800 | 0.34517 | 0.718 | 0.34517 |
| system_71 | RK4 (TP10) | 0.970 | 0.34595 | 0.719 | 0.34595 |
| system_71 | RK4 (TP11) | 0.610 | 0.34721 | 0.721 | 0.34721 |
| system_71 | RK4 (TP12) | 0.940 | 0.34711 | 0.721 | 0.34711 |
| system_71 | RK4 (TP13) | 1.510 | 0.34743 | 0.722 | 0.34743 |
| system_71 | RK4 (TP14) | 2.290 | 0.34757 | 0.722 | 0.34757 |
| system_71 | LA3 (TP8) | 0.320 | 0.34419 | 0.714 | 0.34419 |
| system_71 | LA3 (TP9) | 0.270 | 0.34689 | 0.720 | 0.34689 |
| system_71 | LA3 (TP10) | 0.390 | 0.34737 | 0.721 | 0.34737 |
| system_71 | LA3 (TP11) | 0.570 | 0.34704 | 0.721 | 0.34704 |
| system_71 | LA3 (TP12) | 0.900 | 0.34744 | 0.722 | 0.34744 |
| system_71 | LA3 (TP13) | 1.440 | 0.34753 | 0.722 | 0.34753 |
| system_71 | LA3 (TP14) | 2.220 | 0.34779 | 0.722 | 0.34779 |
| system_71 | LC3 (TP8) | 0.310 | 0.34572 | 0.715 | 0.34572 |
| system_71 | LC3 (TP9) | 0.270 | 0.34545 | 0.715 | 0.34545 |
| system_71 | LC3 (TP10) | 0.400 | 0.34696 | 0.719 | 0.34696 |
| system_71 | LC3 (TP11) | 0.600 | 0.3477 | 0.721 | 0.3477 |
| system_71 | LC3 (TP12) | 0.940 | 0.34745 | 0.721 | 0.34745 |
| system_71 | LC3 (TP13) | 1.490 | 0.34765 | 0.722 | 0.34765 |
| system_71 | LC3 (TP14) | 2.300 | 0.34772 | 0.722 | 0.34772 |
| system_71 | Riot | | | | |
| system_71 | Valencia-IVP (0.00025) | 0m9.028s | 0 | 0.000 | 0 |
| system_71 | Valencia-IVP (0.0025) | 0m0.112s | 0 | 0.000 | 0 |
| system_71 | Valencia-IVP (0.025) | 0m0.007s | 0 | 0.000 | 0 |
| system_71 | VNODE-LP (15, 1e-14,1e-14) | 0m0.028s | 0.093606 | 1.088 | 0.078438 |
| system_71 | VNODE-LP (20, 1e-14,1e-14) | 0m0.035s | 0.094651 | 1.085 | 0.080607 |
| system_71 | VNODE-LP (25, 1e-14,1e-14) | 0m0.034s | 0.095228 | 1.083 | 0.081672 |

Table 3.57: Simulation results of Problem 72

| Problems | Methods | c5t | c5w | c6t | c6w |
|-----------|----------------------------|-----------|------------|--------|------------|
| system_72 | TAYLOR4 (TP8) | 0.110 | 1.888e-08 | 10.000 | 1.5233e-07 |
| system_72 | TAYLOR4 (TP9) | 0.160 | 1.991e-09 | 10.000 | 8.6791e-08 |
| system_72 | TAYLOR4 (TP10) | 0.250 | 2.0536e-10 | 10.000 | 9.4023e-08 |
| system_72 | TAYLOR4 (TP11) | 0.410 | 2.076e-11 | 10.000 | 1.1399e-08 |
| system_72 | TAYLOR4 (TP12) | 0.640 | 2.1992e-12 | 10.000 | 1.6071e-09 |
| system_72 | TAYLOR4 (TP13) | 0.990 | 3.4417e-13 | 10.000 | 2.5738e-10 |
| system_72 | TAYLOR4 (TP14) | 1.550 | 2.3176e-13 | 10.000 | 6.5053e-11 |
| system_72 | RK4 (TP8) | 0.090 | 2.476e-08 | 10.000 | 1.6617e-07 |
| system_72 | RK4 (TP9) | 0.140 | 2.538e-09 | 10.000 | 4.0938e-08 |
| system_72 | RK4 (TP10) | 0.200 | 2.6417e-10 | 10.000 | 5.5074e-08 |
| system_72 | RK4 (TP11) | 0.330 | 2.6895e-11 | 10.000 | 3.9127e-09 |
| system_72 | RK4 (TP12) | 0.520 | 2.7741e-12 | 10.000 | 7.941e-10 |
| system_72 | RK4 (TP13) | 0.800 | 3.2496e-13 | 10.000 | 1.5842e-10 |
| system_72 | RK4 (TP14) | 1.230 | 1.0836e-13 | 10.000 | 3.334e-11 |
| system_72 | LA3 (TP8) | 0.070 | 1.8797e-08 | 10.000 | 1.0346e-06 |
| system_72 | LA3 (TP9) | 0.110 | 1.9786e-09 | 10.000 | 1.3939e-07 |
| system_72 | LA3 (TP10) | 0.160 | 2.0676e-10 | 10.000 | 9.6798e-08 |
| system_72 | LA3 (TP11) | 0.250 | 2.1116e-11 | 10.000 | 1.2856e-08 |
| system_72 | LA3 (TP12) | 0.750 | 2.126e-12 | 10.000 | 1.7966e-09 |
| system_72 | LA3 (TP13) | 0.620 | 2.4913e-13 | 10.000 | 2.5436e-10 |
| system_72 | LA3 (TP14) | 0.950 | 8.1712e-14 | 10.000 | 4.3266e-11 |
| system_72 | LC3 (TP8) | 0.080 | 1.9008e-08 | 10.000 | 3.6404e-06 |
| system_72 | LC3 (TP9) | 0.120 | 2.0101e-09 | 10.000 | 2.7038e-07 |
| system_72 | LC3 (TP10) | 0.180 | 2.0895e-10 | 10.000 | 1.3971e-07 |
| system_72 | LC3 (TP11) | 0.280 | 2.1388e-11 | 10.000 | 2.2361e-08 |
| system_72 | LC3 (TP12) | 0.430 | 2.1335e-12 | 10.000 | 2.1033e-09 |
| system_72 | LC3 (TP13) | 0.690 | 2.5269e-13 | 10.000 | 2.8453e-10 |
| system_72 | LC3 (TP14) | 1.040 | 8.632e-14 | 10.000 | 4.7709e-11 |
| system_72 | Riot (05, 1e-11) | 0m1.648s | 6.8875e-11 | -0.000 | 0.0018269 |
| system_72 | Riot (10, 1e-11) | 0m1.461s | 4.1078e-15 | -0.000 | 7.1333e-13 |
| system_72 | Riot (15, 1e-11) | 0m1.542s | 1.4155e-15 | -0.000 | 9.9245e-15 |
| system_72 | Valencia-IVP (0.00025) | 1m10.076s | 0.011379 | 4.194 | 999.68 |
| system_72 | Valencia-IVP (0.0025) | 0m0.692s | 0.11581 | 3.530 | 992.01 |
| system_72 | Valencia-IVP (0.025) | 0m0.061s | 1.3941 | 2.750 | 956.94 |
| system_72 | VNODE-LP (15, 1e-14,1e-14) | 0m0.014s | 9.1593e-16 | 10.000 | 1.9629e-16 |
| system_72 | VNODE-LP (20, 1e-14,1e-14) | 0m0.010s | 9.1593e-16 | 10.000 | 1.4984e-16 |
| system_72 | VNODE-LP (25, 1e-14,1e-14) | 0m0.010s | 3.8858e-16 | 10.000 | 7.7839e-17 |

Table 3.58: Simulation results of Problem 73

| Problems | Methods | c5t | c5w | c6t | c6w |
|-----------|----------------------------|----------|---------|--------|------------|
| system_73 | TAYLOR4 (TP8) | 0.200 | 0.64903 | 10.000 | 0.00011543 |
| system_73 | TAYLOR4 (TP9) | 0.280 | 0.64903 | 10.000 | 0.00011391 |
| system_73 | TAYLOR4 (TP10) | 0.410 | 0.64903 | 10.000 | 0.00011378 |
| system_73 | TAYLOR4 (TP11) | 0.620 | 0.64903 | 10.000 | 0.00011407 |
| system_73 | TAYLOR4 (TP12) | 1.730 | 0.64903 | 10.000 | 0.00011593 |
| system_73 | TAYLOR4 (TP13) | 1.560 | 0.64903 | 10.000 | 0.00011625 |
| system_73 | TAYLOR4 (TP14) | 2.420 | 0.64903 | 10.000 | 0.00011618 |
| system_73 | RK4 (TP8) | 0.130 | 0.64903 | 10.000 | 0.00011568 |
| system_73 | RK4 (TP9) | 0.200 | 0.64903 | 10.000 | 0.0001142 |
| system_73 | RK4 (TP10) | 0.310 | 0.64903 | 10.000 | 0.00011396 |
| system_73 | RK4 (TP11) | 0.480 | 0.64903 | 10.000 | 0.00011377 |
| system_73 | RK4 (TP12) | 0.760 | 0.64903 | 10.000 | 0.00011457 |
| system_73 | RK4 (TP13) | 1.200 | 0.64903 | 10.000 | 0.00011523 |
| system_73 | RK4 (TP14) | 1.850 | 0.64903 | 10.000 | 0.00011523 |
| system_73 | LA3 (TP8) | 0.110 | 0.90804 | 10.000 | 44.669 |
| system_73 | LA3 (TP9) | 0.160 | 0.84683 | 10.000 | 11.972 |
| system_73 | LA3 (TP10) | 0.240 | 0.74939 | 10.000 | 1.4182 |
| system_73 | LA3 (TP11) | 0.380 | 0.72469 | 10.000 | 2.9375 |
| system_73 | LA3 (TP12) | 0.600 | 0.6851 | 10.000 | 0.3029 |
| system_73 | LA3 (TP13) | 0.980 | 0.67844 | 10.000 | 0.28271 |
| system_73 | LA3 (TP14) | 1.500 | 0.66826 | 10.000 | 0.1337 |
| system_73 | LC3 (TP8) | 0.120 | 0.95006 | 10.000 | 41.555 |
| system_73 | LC3 (TP9) | 0.170 | 0.85351 | 10.000 | 4.46 |
| system_73 | LC3 (TP10) | 0.270 | 0.78623 | 10.000 | 3.7844 |
| system_73 | LC3 (TP11) | 0.420 | 0.72942 | 10.000 | 1.3191 |
| system_73 | LC3 (TP12) | 0.660 | 0.69002 | 10.000 | 0.35275 |
| system_73 | LC3 (TP13) | 1.060 | 0.67725 | 10.000 | 0.3206 |
| system_73 | LC3 (TP14) | 1.660 | 0.66841 | 10.000 | 0.061096 |
| system_73 | Riot (05, 1e-11) | 0m1.815s | 0.64903 | -0.000 | 0.00011995 |
| system_73 | Riot (10, 1e-11) | 0m2.136s | 0.64903 | -0.000 | 0.0001136 |
| system_73 | Riot (15, 1e-11) | 0m3.216s | 0.64903 | -0.000 | 0.00011366 |
| system_73 | Valencia-IVP (0.00025) | 1m1.164s | 138.84 | 1.367 | 999.11 |
| system_73 | Valencia-IVP (0.0025) | 0m0.278s | 145.77 | 1.355 | 994.94 |
| system_73 | Valencia-IVP (0.025) | 0m0.029s | 243.46 | 1.225 | 891.88 |
| system_73 | VNODE-LP (15, 1e-14,1e-14) | 0m0.024s | 0.64903 | 10.000 | 3.4442 |
| system_73 | VNODE-LP (20, 1e-14,1e-14) | 0m0.015s | 0.64903 | 10.000 | 3.4442 |
| system_73 | VNODE-LP (25, 1e-14,1e-14) | 0m0.016s | 0.64903 | 10.000 | 3.4442 |

Table 3.59: Simulation results of Problem 74

| Problems | Methods | c5t | c5w | c6t | c6w |
|-----------|----------------------------|----------|------------|-------|------------|
| system_74 | TAYLOR4 (TP8) | 0.250 | 430.91 | 0.785 | 430.91 |
| system_74 | TAYLOR4 (TP9) | 0.350 | 652.41 | 0.785 | 652.41 |
| system_74 | TAYLOR4 (TP10) | 0.490 | 283.48 | 0.785 | 283.48 |
| system_74 | TAYLOR4 (TP11) | 0.670 | 559.65 | 0.785 | 559.65 |
| system_74 | TAYLOR4 (TP12) | 0.890 | 576.25 | 0.785 | 576.25 |
| system_74 | TAYLOR4 (TP13) | 1.290 | 234.53 | 0.785 | 234.53 |
| system_74 | TAYLOR4 (TP14) | 0.000 | 0 | 0.000 | 0 |
| system_74 | RK4 (TP8) | 0.130 | 624.24 | 0.785 | 624.24 |
| system_74 | RK4 (TP9) | 0.160 | 57.925 | 0.785 | 57.925 |
| system_74 | RK4 (TP10) | 0.220 | 330.01 | 0.785 | 330.01 |
| system_74 | RK4 (TP11) | 0.310 | 268.64 | 0.785 | 268.64 |
| system_74 | RK4 (TP12) | 0.440 | 44.208 | 0.785 | 44.208 |
| system_74 | RK4 (TP13) | 0.620 | 267.16 | 0.785 | 267.16 |
| system_74 | RK4 (TP14) | 0.860 | 74.118 | 0.785 | 74.118 |
| system_74 | LA3 (TP8) | 0.130 | 76.095 | 0.785 | 76.095 |
| system_74 | LA3 (TP9) | 0.190 | 45.448 | 0.785 | 45.448 |
| system_74 | LA3 (TP10) | 0.260 | 62.95 | 0.785 | 62.95 |
| system_74 | LA3 (TP11) | 0.370 | 64.448 | 0.785 | 64.448 |
| system_74 | LA3 (TP12) | 0.530 | 527.45 | 0.785 | 527.45 |
| system_74 | LA3 (TP13) | 0.730 | 21.878 | 0.785 | 21.878 |
| system_74 | LA3 (TP14) | 1.040 | 266.61 | 0.785 | 266.61 |
| system_74 | LC3 (TP8) | 0.100 | 90.528 | 0.785 | 90.528 |
| system_74 | LC3 (TP9) | 0.130 | 61.895 | 0.785 | 61.895 |
| system_74 | LC3 (TP10) | 0.170 | 79.971 | 0.785 | 79.971 |
| system_74 | LC3 (TP11) | 0.240 | 104.2 | 0.785 | 104.2 |
| system_74 | LC3 (TP12) | 0.320 | 8.7342 | 0.785 | 8.7342 |
| system_74 | LC3 (TP13) | 0.450 | 205.63 | 0.785 | 205.63 |
| system_74 | LC3 (TP14) | 0.640 | 258.77 | 0.785 | 258.77 |
| system_74 | Riot (05, 1e-11) | 0m0.791s | 0 | 0.000 | 0 |
| system_74 | Riot (10, 1e-11) | 0m0.430s | 0 | 0.000 | 0 |
| system_74 | Riot (15, 1e-11) | 0m0.613s | 0 | 0.000 | 0 |
| system_74 | Valencia-IVP (0.00025) | 0m9.104s | 668.07 | 0.783 | 668.07 |
| system_74 | Valencia-IVP (0.0025) | 0m0.165s | 60.454 | 0.765 | 60.454 |
| system_74 | Valencia-IVP (0.025) | 0m0.014s | 5.325 | 0.650 | 5.325 |
| system_74 | VNODE-LP (15, 1e-14,1e-14) | 0m0.014s | 4992.7 | 0.015 | 4992.7 |
| system_74 | VNODE-LP (20, 1e-14,1e-14) | 0m0.023s | 2.2247e-07 | 0.785 | 2.2247e-07 |
| system_74 | VNODE-LP (25, 1e-14,1e-14) | 0m0.010s | 16182 | 0.001 | 16182 |

3.1.3 Discussion

Firstly, we count the number of problem for which each method (for each order and each precision) is first in term of solution diameter, second or last. This account is done for the simulation at 1 second and at 10 seconds. The results are summarized in the table 3.60. Of course, we are aware that the results are biased by the number of methods we have. Nevertheless, this table allows us to consider that Valencia and Riot are not valid competitors.

Table 3.60: Number of times a method produced the sharpest enclosure or the second sharpest enclosure.

| Method | c5w (1st) | c5w (2nd) | c5w (last) | c6w (1st) | c6w (2nd) | c6w (last) |
|----------|--------------|--------------|---------------|--------------|--------------|---------------|
| RK | 103 | 35 | 8 | 58 | 39 | 8 |
| Vnode-LP | 70 | 28 | 9 | 44 | 27 | 8 |
| Riot | 36 | 11 | 0 | 24 | 12 | 2 |
| Valencia | 3 | 3 | 49 | 3 | 2 | 49 |

After this reduction of competitors, only the best results for our three order-4 Runge-Kutta methods, and for Vnode are kept for comparison. We present in the spider graph 3.1, respectively 3.2, the normalized results (divided by the median and multiplied by 10) for each problem for a simulation at 1 second, respectively at 10 seconds. The median used to normalize the results is computed with all the methods: Taylor4, RK4, LA3, LC3 Riot, Valencia and Vnode (for all precision and all order).

Remark: for the graph 3.1, we truncate the results at 25 for the clarity. It leads to the truncation of LC3 result for problem 44, initially at 178, the result is set at 25. In the same manner, the results are also truncated at 50 for the graph 3.2, fifteen times for Vnode, one time for LC3 and one time for RK4.

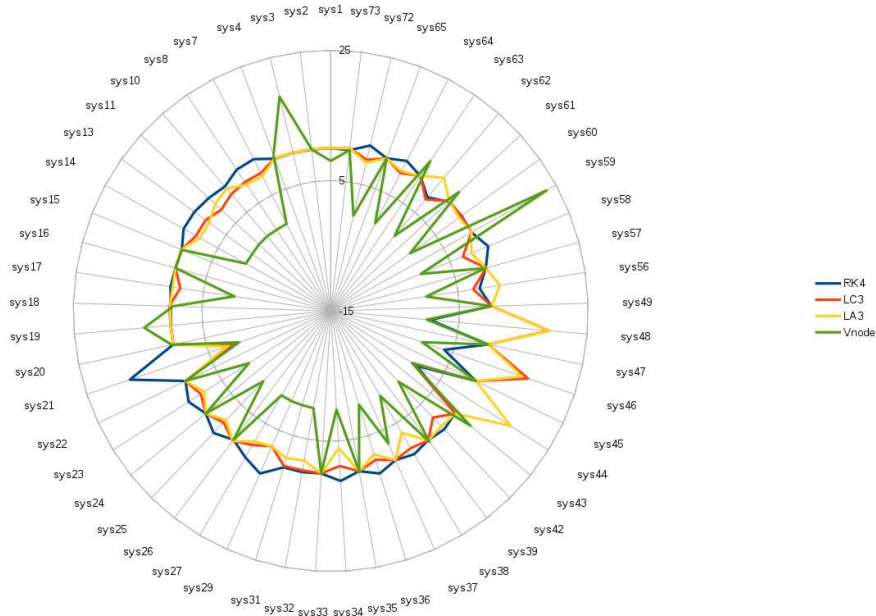


Figure 3.1: Results gathered in spider graph for a simulation of 1 second, for the methods: RK4, LC3, LA3 and Vnode

We can easily see on spider graph 3.1 that the Runge-Kutta methods are more stable, by describing a circle while Vnode results are more in a star shape. Moreover, the implicit methods (LA3 and LC3) provide better results than the explicit RK4 in a majority of problems. This fact is even more clear on the graph 3.2. On this latter graph, we can also see that Vnode fails many times while at least one of our Runge-Kutta methods performs a good simulation for all the considered problems. Finally, if Vnode are the best on many problems, by our stability and our better results for some problems, we can conclude

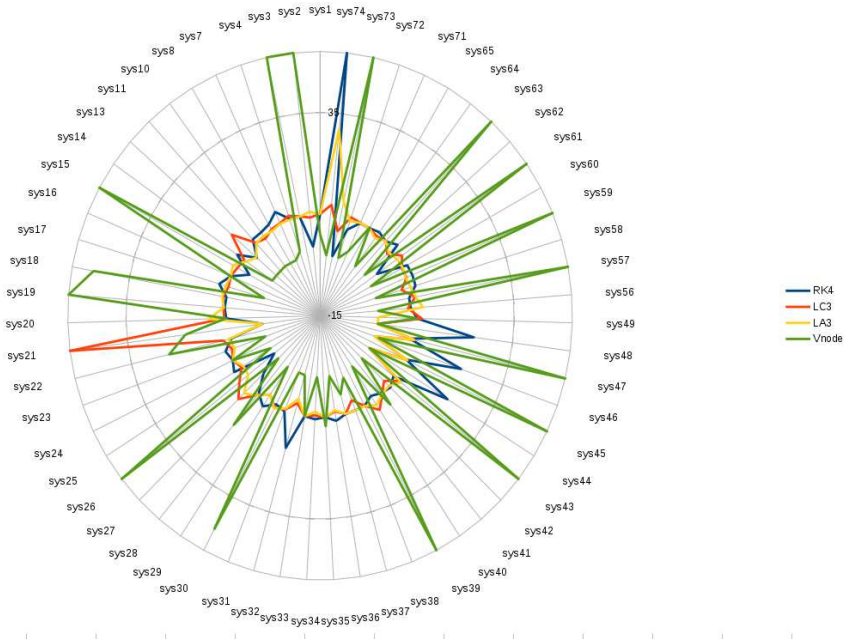


Figure 3.2: Results gathered in spider graph for a simulation of 10 seconds, for the methods: RK4, LC3, LA3 and Vnode

that our tool is a good competitor for Vnode. The last remark but not the least, it is important to remember that we have currently only methods of order 4, when Vnode can use a Taylor at order 25 !

3.2 Detest benchmark

3.2.1 Disclaimer

This section reports the results of the solution of various problems coming from the DETEST benchmark. For each problem, different validated methods of Runge-Kutta of order 4 are applied among: the classical formula of Runge-Kutta (explicit), the Lobatto-3a formula (implicit) and the Lobatto-3c formula (implicit). Moreover, an homemade version of Taylor series, limited to order 5 and using affine arithmetic, is also applied on each problem.

For each problem, we report the following metrics:

- c5t: user time taken to simulate the problem for 1 second.
- c5w: the final diameter of the solution (infinity norm is used).
- c6t: the time to breakdown the method with a maximal limit of 10 seconds.
- c6w: the diameter of the solution a the breakdown time.

3.2.2 Results

Table 3.61: Simulation results of Problem ns_A1

| Problems | Methods | c5t | c5w | c6t | c6w |
|----------|----------------|-------|------------|--------|------------|
| ns_A1 | TAYLOR4 (TP4) | 0.030 | 9.146e-06 | 10.000 | 6.3861e-06 |
| ns_A1 | TAYLOR4 (TP6) | 0.030 | 5.0222e-07 | 2.000 | 9.7332e-07 |
| ns_A1 | TAYLOR4 (TP8) | 0.060 | 6.0636e-09 | 2.000 | 5.7233e-08 |
| ns_A1 | TAYLOR4 (TP10) | 0.120 | 6.3146e-11 | 2.000 | 6.7023e-10 |
| ns_A1 | TAYLOR4 (TP12) | 0.300 | 7.1687e-13 | 10.000 | 5.5133e-12 |
| ns_A1 | TAYLOR4 (TP14) | 0.020 | 9.146e-06 | 10.000 | 6.3861e-06 |
| ns_A1 | RK4 (TP4) | 0.010 | 9.146e-06 | 10.000 | 6.2632e-06 |
| ns_A1 | RK4 (TP6) | 0.020 | 7.1338e-07 | 2.000 | 1.236e-06 |
| ns_A1 | RK4 (TP8) | 0.030 | 7.4993e-09 | 2.000 | 4.3775e-08 |
| ns_A1 | RK4 (TP10) | 0.060 | 8.4251e-11 | 2.000 | 6.7118e-10 |
| ns_A1 | RK4 (TP12) | 0.160 | 8.8185e-13 | 10.000 | 7.5966e-12 |
| ns_A1 | RK4 (TP14) | 0.010 | 9.146e-06 | 10.000 | 6.2632e-06 |
| ns_A1 | LA3 (TP4) | 0.010 | 1.531e-06 | 10.000 | 7.6554e-06 |
| ns_A1 | LA3 (TP6) | 0.020 | 4.0741e-07 | 2.000 | 8.1525e-07 |
| ns_A1 | LA3 (TP8) | 0.020 | 5.4981e-09 | 2.000 | 4.1256e-08 |
| ns_A1 | LA3 (TP10) | 0.050 | 6.1542e-11 | 2.000 | 5.8395e-10 |
| ns_A1 | LA3 (TP12) | 0.130 | 6.7724e-13 | 10.000 | 5.3249e-12 |
| ns_A1 | LA3 (TP14) | 0.010 | 1.531e-06 | 10.000 | 7.6554e-06 |
| ns_A1 | LC3 (TP4) | 0.010 | 2.3003e-06 | 10.000 | 7.8708e-05 |
| ns_A1 | LC3 (TP6) | 0.020 | 3.8815e-07 | 2.000 | 1.1053e-06 |
| ns_A1 | LC3 (TP8) | 0.030 | 5.8283e-09 | 2.000 | 4.7752e-08 |
| ns_A1 | LC3 (TP10) | 0.060 | 6.1916e-11 | 2.000 | 6.2382e-10 |
| ns_A1 | LC3 (TP12) | 0.140 | 6.7468e-13 | 10.000 | 5.3717e-12 |
| ns_A1 | LC3 (TP14) | 0.020 | 2.3003e-06 | 10.000 | 7.8708e-05 |

Table 3.62: Simulation results of Problem ns_A2

| Problems | Methods | c5t | c5w | c6t | c6w |
|----------|----------------|-------|------------|--------|------------|
| ns_A2 | TAYLOR4 (TP4) | 0.040 | 8.4667e-05 | 10.000 | 8.3982e-05 |
| ns_A2 | TAYLOR4 (TP6) | 0.050 | 1.5064e-06 | 2.000 | 3.1707e-06 |
| ns_A2 | TAYLOR4 (TP8) | 0.080 | 2.1535e-08 | 2.000 | 1.4215e-07 |
| ns_A2 | TAYLOR4 (TP10) | 0.180 | 2.4119e-10 | 2.000 | 1.6527e-09 |
| ns_A2 | TAYLOR4 (TP12) | 0.440 | 2.6019e-12 | 10.000 | 1.3628e-11 |
| ns_A2 | TAYLOR4 (TP14) | 0.040 | 8.4667e-05 | 10.000 | 8.3982e-05 |
| ns_A2 | RK4 (TP4) | 0.020 | 1.6376e-05 | 10.000 | 4.3081e-05 |
| ns_A2 | RK4 (TP6) | 0.030 | 2.3535e-06 | 2.000 | 4.883e-06 |
| ns_A2 | RK4 (TP8) | 0.040 | 4.9213e-08 | 2.000 | 2.9883e-07 |
| ns_A2 | RK4 (TP10) | 0.060 | 5.2365e-10 | 2.000 | 3.509e-09 |
| ns_A2 | RK4 (TP12) | 0.140 | 4.6034e-12 | 10.000 | 2.7551e-11 |
| ns_A2 | RK4 (TP14) | 0.020 | 1.6376e-05 | 10.000 | 4.3081e-05 |
| ns_A2 | LA3 (TP4) | 0.020 | 9.2675e-06 | 10.000 | 2.3649e-05 |
| ns_A2 | LA3 (TP6) | 0.030 | 1.1342e-06 | 2.000 | 3.1789e-06 |
| ns_A2 | LA3 (TP8) | 0.040 | 1.9817e-08 | 2.000 | 1.116e-07 |
| ns_A2 | LA3 (TP10) | 0.080 | 2.3168e-10 | 2.000 | 1.5203e-09 |
| ns_A2 | LA3 (TP12) | 0.200 | 2.5466e-12 | 10.000 | 1.3165e-11 |
| ns_A2 | LA3 (TP14) | 0.020 | 9.2675e-06 | 10.000 | 2.3649e-05 |
| ns_A2 | LC3 (TP4) | 0.020 | 7.5652e-06 | 10.000 | 2.6357e-05 |
| ns_A2 | LC3 (TP6) | 0.030 | 1.4833e-06 | 2.000 | 3.5055e-06 |
| ns_A2 | LC3 (TP8) | 0.040 | 2.6328e-08 | 2.000 | 1.5975e-07 |
| ns_A2 | LC3 (TP10) | 0.070 | 2.9172e-10 | 2.000 | 1.9148e-09 |
| ns_A2 | LC3 (TP12) | 0.150 | 2.828e-12 | 10.000 | 1.5799e-11 |
| ns_A2 | LC3 (TP14) | 0.020 | 7.5652e-06 | 10.000 | 2.6357e-05 |

Table 3.63: Simulation results of Problem ns_A3

| Problems | Methods | c5t | c5w | c6t | c6w |
|----------|----------------|-------|------------|--------|------------|
| ns_A3 | TAYLOR4 (TP4) | 0.050 | 0.00043573 | 10.000 | 0.0041836 |
| ns_A3 | TAYLOR4 (TP6) | 0.060 | 8.3465e-06 | 2.000 | 1.6766e-05 |
| ns_A3 | TAYLOR4 (TP8) | 0.110 | 1.0131e-07 | 2.000 | 2.4257e-07 |
| ns_A3 | TAYLOR4 (TP10) | 0.220 | 1.5521e-09 | 2.000 | 3.3297e-09 |
| ns_A3 | TAYLOR4 (TP12) | 0.550 | 1.9743e-11 | 10.000 | 1.544e-10 |
| ns_A3 | TAYLOR4 (TP14) | 0.050 | 0.00043573 | 10.000 | 0.0041836 |
| ns_A3 | RK4 (TP4) | 0.030 | 0.00014736 | 10.000 | 0.004336 |
| ns_A3 | RK4 (TP6) | 0.040 | 8.2963e-06 | 2.000 | 2.5968e-05 |
| ns_A3 | RK4 (TP8) | 0.050 | 1.1569e-07 | 2.000 | 5.2775e-07 |
| ns_A3 | RK4 (TP10) | 0.120 | 1.4826e-09 | 2.000 | 1.0182e-08 |
| ns_A3 | RK4 (TP12) | 0.260 | 1.6631e-11 | 10.000 | 4.1913e-10 |
| ns_A3 | RK4 (TP14) | 0.030 | 0.00014736 | 10.000 | 0.004336 |
| ns_A3 | LA3 (TP4) | 0.030 | 7.0869e-05 | 10.000 | 0.0049959 |
| ns_A3 | LA3 (TP6) | 0.040 | 4.0701e-06 | 2.000 | 9.8443e-06 |
| ns_A3 | LA3 (TP8) | 0.060 | 4.8721e-08 | 2.000 | 1.7833e-07 |
| ns_A3 | LA3 (TP10) | 0.130 | 5.5251e-10 | 2.000 | 2.0766e-09 |
| ns_A3 | LA3 (TP12) | 0.310 | 5.917e-12 | 10.000 | 1.2948e-10 |
| ns_A3 | LA3 (TP14) | 0.030 | 7.0869e-05 | 10.000 | 0.0049959 |
| ns_A3 | LC3 (TP4) | 0.030 | 8.4934e-05 | 10.000 | 0.0056809 |
| ns_A3 | LC3 (TP6) | 0.030 | 8.2958e-06 | 2.000 | 1.9432e-05 |
| ns_A3 | LC3 (TP8) | 0.060 | 8.8322e-08 | 2.000 | 2.3749e-07 |
| ns_A3 | LC3 (TP10) | 0.110 | 1.3559e-09 | 2.000 | 3.5262e-09 |
| ns_A3 | LC3 (TP12) | 0.250 | 1.6706e-11 | 10.000 | 1.5879e-10 |
| ns_A3 | LC3 (TP14) | 0.030 | 8.4934e-05 | 10.000 | 0.0056809 |

Table 3.64: Simulation results of Problem ns_A4

| Problems | Methods | c5t | c5w | c6t | c6w |
|----------|----------------|-------|------------|--------|------------|
| ns_A4 | TAYLOR4 (TP4) | 0.030 | 4.7235e-09 | 10.000 | 5.5164e-05 |
| ns_A4 | TAYLOR4 (TP6) | 0.030 | 4.7235e-09 | 10.000 | 2.3393e-05 |
| ns_A4 | TAYLOR4 (TP8) | 0.030 | 4.7235e-09 | 10.000 | 3.347e-07 |
| ns_A4 | TAYLOR4 (TP10) | 0.040 | 1.0831e-10 | 10.000 | 4.3438e-09 |
| ns_A4 | TAYLOR4 (TP12) | 0.080 | 1.394e-12 | 10.000 | 5.5834e-11 |
| ns_A4 | TAYLOR4 (TP14) | 0.030 | 4.7235e-09 | 10.000 | 5.5164e-05 |
| ns_A4 | RK4 (TP4) | 0.020 | 1.9557e-09 | 10.000 | 9.2536e-06 |
| ns_A4 | RK4 (TP6) | 0.020 | 1.9557e-09 | 10.000 | 7.5759e-06 |
| ns_A4 | RK4 (TP8) | 0.020 | 1.4271e-09 | 10.000 | 1.0095e-07 |
| ns_A4 | RK4 (TP10) | 0.030 | 2.1419e-11 | 10.000 | 1.0543e-09 |
| ns_A4 | RK4 (TP12) | 0.060 | 2.9976e-13 | 10.000 | 1.1743e-11 |
| ns_A4 | RK4 (TP14) | 0.020 | 1.9557e-09 | 10.000 | 9.2536e-06 |
| ns_A4 | LA3 (TP4) | 0.020 | 8.073e-11 | 10.000 | 3.1954e-06 |
| ns_A4 | LA3 (TP6) | 0.020 | 8.073e-11 | 10.000 | 3.1954e-06 |
| ns_A4 | LA3 (TP8) | 0.020 | 8.073e-11 | 10.000 | 2.7486e-07 |
| ns_A4 | LA3 (TP10) | 0.020 | 5.8513e-12 | 10.000 | 3.7748e-09 |
| ns_A4 | LA3 (TP12) | 0.040 | 7.7716e-14 | 10.000 | 4.1114e-11 |
| ns_A4 | LA3 (TP14) | 0.020 | 8.073e-11 | 10.000 | 3.1954e-06 |
| ns_A4 | LC3 (TP4) | 0.020 | 4.5581e-10 | 10.000 | 4.0216e-06 |
| ns_A4 | LC3 (TP6) | 0.020 | 4.5581e-10 | 10.000 | 4.0216e-06 |
| ns_A4 | LC3 (TP8) | 0.020 | 4.5581e-10 | 10.000 | 2.0247e-07 |
| ns_A4 | LC3 (TP10) | 0.030 | 1.517e-11 | 10.000 | 3.328e-09 |
| ns_A4 | LC3 (TP12) | 0.050 | 2.0917e-13 | 10.000 | 4.2366e-11 |
| ns_A4 | LC3 (TP14) | 0.020 | 4.5581e-10 | 10.000 | 4.0216e-06 |

Table 3.65: Simulation results of Problem ns_A5

| Problems | Methods | c5t | c5w | c6t | c6w |
|----------|----------------|-------|------------|--------|------------|
| ns_A5 | TAYLOR4 (TP4) | 0.070 | 3.7194e-05 | 10.000 | 0.0019729 |
| ns_A5 | TAYLOR4 (TP6) | 0.070 | 3.7194e-05 | 10.000 | 0.00023788 |
| ns_A5 | TAYLOR4 (TP8) | 0.090 | 3.5949e-05 | 10.000 | 0.00011626 |
| ns_A5 | TAYLOR4 (TP10) | 0.140 | 3.5909e-05 | 10.000 | 0.00011476 |
| ns_A5 | TAYLOR4 (TP12) | 0.310 | 3.5909e-05 | 10.000 | 0.00011475 |
| ns_A5 | TAYLOR4 (TP14) | 0.070 | 3.7194e-05 | 10.000 | 0.0019729 |
| ns_A5 | RK4 (TP4) | 0.060 | 1.9565e-05 | 10.000 | 0.00017718 |
| ns_A5 | RK4 (TP6) | 0.050 | 1.9565e-05 | 10.000 | 0.00013889 |
| ns_A5 | RK4 (TP8) | 0.060 | 1.9565e-05 | 10.000 | 6.419e-05 |
| ns_A5 | RK4 (TP10) | 0.070 | 1.9502e-05 | 10.000 | 6.2343e-05 |
| ns_A5 | RK4 (TP12) | 0.120 | 1.9501e-05 | 10.000 | 6.2318e-05 |
| ns_A5 | RK4 (TP14) | 0.060 | 1.9565e-05 | 10.000 | 0.00017718 |
| ns_A5 | LA3 (TP4) | 0.060 | 1.2266e-05 | 10.000 | 0.0001074 |
| ns_A5 | LA3 (TP6) | 0.060 | 1.2266e-05 | 10.000 | 8.3944e-05 |
| ns_A5 | LA3 (TP8) | 0.060 | 1.2241e-05 | 10.000 | 4.0144e-05 |
| ns_A5 | LA3 (TP10) | 0.080 | 1.2208e-05 | 10.000 | 3.9021e-05 |
| ns_A5 | LA3 (TP12) | 0.150 | 1.2208e-05 | 10.000 | 3.9011e-05 |
| ns_A5 | LA3 (TP14) | 0.060 | 1.2266e-05 | 10.000 | 0.0001074 |
| ns_A5 | LC3 (TP4) | 0.060 | 3.0021e-06 | 10.000 | 8.1197e-05 |
| ns_A5 | LC3 (TP6) | 0.060 | 3.0021e-06 | 10.000 | 5.8567e-05 |
| ns_A5 | LC3 (TP8) | 0.060 | 2.9819e-06 | 10.000 | 1.0499e-05 |
| ns_A5 | LC3 (TP10) | 0.080 | 2.9478e-06 | 10.000 | 9.4283e-06 |
| ns_A5 | LC3 (TP12) | 0.140 | 2.9474e-06 | 10.000 | 9.4188e-06 |
| ns_A5 | LC3 (TP14) | 0.060 | 3.0021e-06 | 10.000 | 8.1197e-05 |

Table 3.66: Simulation results of Problem ns_B1

| Problems | Methods | c5t | c5w | c6t | c6w |
|----------|----------------|-------|------------|--------|------------|
| ns_B1 | TAYLOR4 (TP4) | 0.120 | 0.00041929 | 10.000 | 0.16516 |
| ns_B1 | TAYLOR4 (TP6) | 0.180 | 5.8337e-06 | 2.000 | 8.016e-05 |
| ns_B1 | TAYLOR4 (TP8) | 0.270 | 1.5364e-07 | 2.000 | 3.7536e-05 |
| ns_B1 | TAYLOR4 (TP10) | 0.610 | 1.6928e-09 | 2.000 | 1.2351e-05 |
| ns_B1 | TAYLOR4 (TP12) | 1.500 | 1.847e-11 | 10.000 | 7.9824e-07 |
| ns_B1 | TAYLOR4 (TP14) | 0.120 | 0.00041929 | 10.000 | 0.16516 |
| ns_B1 | RK4 (TP4) | 0.060 | 0.00054791 | 10.000 | 0.093055 |
| ns_B1 | RK4 (TP6) | 0.090 | 7.7186e-06 | 2.000 | 7.0418e-05 |
| ns_B1 | RK4 (TP8) | 0.160 | 1.4924e-07 | 2.000 | 8.8254e-06 |
| ns_B1 | RK4 (TP10) | 0.370 | 1.6512e-09 | 2.000 | 3.6011e-06 |
| ns_B1 | RK4 (TP12) | 1.260 | 1.7231e-11 | 10.000 | 4.5013e-07 |
| ns_B1 | RK4 (TP14) | 0.070 | 0.00054791 | 10.000 | 0.093055 |
| ns_B1 | LA3 (TP4) | 0.060 | 0.00052296 | 10.000 | 0.7639 |
| ns_B1 | LA3 (TP6) | 0.090 | 5.981e-06 | 2.000 | 6.7454e-05 |
| ns_B1 | LA3 (TP8) | 0.150 | 1.3016e-07 | 2.000 | 3.6223e-05 |
| ns_B1 | LA3 (TP10) | 0.380 | 1.5537e-09 | 2.000 | 1.2472e-05 |
| ns_B1 | LA3 (TP12) | 0.820 | 1.5877e-11 | 10.000 | 4.2412e-07 |
| ns_B1 | LA3 (TP14) | 0.060 | 0.00052296 | 10.000 | 0.7639 |
| ns_B1 | LC3 (TP4) | 0.070 | 0.00074279 | 10.000 | 7.958 |
| ns_B1 | LC3 (TP6) | 0.080 | 8.5157e-06 | 2.000 | 0.00010335 |
| ns_B1 | LC3 (TP8) | 0.160 | 1.2294e-07 | 2.000 | 3.5055e-05 |
| ns_B1 | LC3 (TP10) | 0.330 | 1.1696e-09 | 2.000 | 5.7342e-06 |
| ns_B1 | LC3 (TP12) | 0.770 | 1.1289e-11 | 10.000 | 4.7543e-07 |
| ns_B1 | LC3 (TP14) | 0.070 | 0.00074279 | 10.000 | 7.958 |

Table 3.67: Simulation results of Problem ns_B2

| Problems | Methods | c5t | c5w | c6t | c6w |
|----------|----------------|-------|------------|--------|------------|
| ns_B2 | TAYLOR4 (TP4) | 0.310 | 8.7614e-05 | 10.000 | 0.00010474 |
| ns_B2 | TAYLOR4 (TP6) | 0.540 | 1.0578e-06 | 2.000 | 7.0248e-06 |
| ns_B2 | TAYLOR4 (TP8) | 1.030 | 2.3614e-08 | 2.000 | 4.1597e-06 |
| ns_B2 | TAYLOR4 (TP10) | 2.470 | 2.5418e-10 | 2.000 | 4.92e-08 |
| ns_B2 | TAYLOR4 (TP12) | 6.170 | 2.8764e-12 | 10.000 | 2.2351e-10 |
| ns_B2 | TAYLOR4 (TP14) | 0.330 | 8.7614e-05 | 10.000 | 0.00010474 |
| ns_B2 | RK4 (TP4) | 0.200 | 9.85e-05 | 10.000 | 0.00014666 |
| ns_B2 | RK4 (TP6) | 0.320 | 1.4878e-06 | 2.000 | 5.1326e-06 |
| ns_B2 | RK4 (TP8) | 0.630 | 2.8479e-08 | 2.000 | 2.8268e-06 |
| ns_B2 | RK4 (TP10) | 1.510 | 3.244e-10 | 2.000 | 5.9749e-08 |
| ns_B2 | RK4 (TP12) | 3.710 | 3.4948e-12 | 10.000 | 1.4806e-10 |
| ns_B2 | RK4 (TP14) | 0.200 | 9.85e-05 | 10.000 | 0.00014666 |
| ns_B2 | LA3 (TP4) | 0.210 | 0.00011841 | 10.000 | 0.049815 |
| ns_B2 | LA3 (TP6) | 0.270 | 1.0755e-06 | 2.000 | 8.8929e-06 |
| ns_B2 | LA3 (TP8) | 0.490 | 2.1817e-08 | 2.000 | 3.4971e-06 |
| ns_B2 | LA3 (TP10) | 1.110 | 2.4909e-10 | 2.000 | 4.7146e-08 |
| ns_B2 | LA3 (TP12) | 2.830 | 2.6863e-12 | 10.000 | 1.7301e-10 |
| ns_B2 | LA3 (TP14) | 0.200 | 0.00011841 | 10.000 | 0.049815 |
| ns_B2 | LC3 (TP4) | 0.210 | 0.00011385 | 10.000 | 0.11981 |
| ns_B2 | LC3 (TP6) | 0.290 | 1.2619e-06 | 2.000 | 1.129e-05 |
| ns_B2 | LC3 (TP8) | 0.540 | 2.2956e-08 | 2.000 | 4.0489e-06 |
| ns_B2 | LC3 (TP10) | 1.240 | 2.5586e-10 | 2.000 | 4.9842e-08 |
| ns_B2 | LC3 (TP12) | 3.060 | 2.7098e-12 | 10.000 | 2.22e-10 |
| ns_B2 | LC3 (TP14) | 0.200 | 0.00011385 | 10.000 | 0.11981 |

Table 3.68: Simulation results of Problem ns_B3

| Problems | Methods | c5t | c5w | c6t | c6w |
|----------|----------------|-------|------------|--------|------------|
| ns_B3 | TAYLOR4 (TP4) | 0.240 | 0.00012496 | 10.000 | 0.00010291 |
| ns_B3 | TAYLOR4 (TP6) | 0.330 | 2.3385e-06 | 2.000 | 5.5216e-06 |
| ns_B3 | TAYLOR4 (TP8) | 0.640 | 2.7922e-08 | 2.000 | 2.5563e-07 |
| ns_B3 | TAYLOR4 (TP10) | 1.400 | 4.1428e-10 | 2.000 | 4.1314e-09 |
| ns_B3 | TAYLOR4 (TP12) | 3.370 | 5.4439e-12 | 10.000 | 8.7434e-12 |
| ns_B3 | TAYLOR4 (TP14) | 0.230 | 0.00012496 | 10.000 | 0.00010291 |
| ns_B3 | RK4 (TP4) | 0.140 | 0.00015668 | 10.000 | 3.2946e-05 |
| ns_B3 | RK4 (TP6) | 0.200 | 1.8062e-06 | 2.000 | 3.3727e-06 |
| ns_B3 | RK4 (TP8) | 0.380 | 2.132e-08 | 2.000 | 1.8169e-07 |
| ns_B3 | RK4 (TP10) | 0.830 | 2.2347e-10 | 2.000 | 1.9818e-09 |
| ns_B3 | RK4 (TP12) | 2.050 | 2.2799e-12 | 10.000 | 5.204e-12 |
| ns_B3 | RK4 (TP14) | 0.140 | 0.00015668 | 10.000 | 3.2946e-05 |
| ns_B3 | LA3 (TP4) | 0.140 | 4.8032e-05 | 10.000 | 4.5514e-05 |
| ns_B3 | LA3 (TP6) | 0.170 | 2.1365e-06 | 2.000 | 4.9673e-06 |
| ns_B3 | LA3 (TP8) | 0.260 | 2.3226e-08 | 2.000 | 2.0047e-07 |
| ns_B3 | LA3 (TP10) | 0.540 | 1.8545e-10 | 2.000 | 1.8394e-09 |
| ns_B3 | LA3 (TP12) | 1.220 | 1.6824e-12 | 10.000 | 6.1119e-12 |
| ns_B3 | LA3 (TP14) | 0.150 | 4.8032e-05 | 10.000 | 4.5514e-05 |
| ns_B3 | LC3 (TP4) | 0.150 | 7.7124e-05 | 10.000 | 8.204e-05 |
| ns_B3 | LC3 (TP6) | 0.210 | 1.6073e-06 | 2.000 | 3.8007e-06 |
| ns_B3 | LC3 (TP8) | 0.290 | 3.1901e-08 | 2.000 | 2.6442e-07 |
| ns_B3 | LC3 (TP10) | 0.590 | 3.4737e-10 | 2.000 | 3.3251e-09 |
| ns_B3 | LC3 (TP12) | 1.410 | 3.5121e-12 | 10.000 | 3.7007e-12 |
| ns_B3 | LC3 (TP14) | 0.140 | 7.7124e-05 | 10.000 | 8.204e-05 |

Table 3.69: Simulation results of Problem ns_B4

| Problems | Methods | c5t | c5w | c6t | c6w |
|----------|----------------|---------|------------|--------|------------|
| ns_B4 | TAYLOR4 (TP4) | 17.290 | 0.0016863 | 4.340 | 0.90148 |
| ns_B4 | TAYLOR4 (TP6) | 19.640 | 8.0895e-05 | 7.751 | 1.2422 |
| ns_B4 | TAYLOR4 (TP8) | 27.690 | 1.5611e-06 | 9.522 | 0.62151 |
| ns_B4 | TAYLOR4 (TP10) | 54.410 | 2.2374e-08 | 10.000 | 0.029197 |
| ns_B4 | TAYLOR4 (TP12) | 123.820 | 2.9517e-10 | 10.000 | 0.0011596 |
| ns_B4 | TAYLOR4 (TP14) | 17.280 | 0.0016863 | 4.340 | 0.90148 |
| ns_B4 | RK4 (TP4) | 16.010 | 0.00053691 | 5.537 | 1.439 |
| ns_B4 | RK4 (TP6) | 17.720 | 2.7416e-05 | 8.560 | 0.69971 |
| ns_B4 | RK4 (TP8) | 19.930 | 1.0075e-06 | 10.000 | 0.54239 |
| ns_B4 | RK4 (TP10) | 33.190 | 1.5332e-08 | 10.000 | 0.023281 |
| ns_B4 | RK4 (TP12) | 70.370 | 1.8093e-10 | 10.000 | 0.00066257 |
| ns_B4 | RK4 (TP14) | 16.010 | 0.00053691 | 5.537 | 1.439 |
| ns_B4 | LA3 (TP4) | 15.970 | 0.00044803 | 3.505 | 0.031706 |
| ns_B4 | LA3 (TP6) | 17.740 | 2.2201e-05 | 8.281 | 0.86537 |
| ns_B4 | LA3 (TP8) | 17.980 | 1.1266e-06 | 10.000 | 0.60409 |
| ns_B4 | LA3 (TP10) | 31.530 | 1.6075e-08 | 10.000 | 0.026108 |
| ns_B4 | LA3 (TP12) | 65.940 | 2.1847e-10 | 10.000 | 0.00083967 |
| ns_B4 | LA3 (TP14) | 15.940 | 0.00044803 | 3.505 | 0.031706 |
| ns_B4 | LC3 (TP4) | 14.510 | 0.00097031 | 4.842 | 1.322 |
| ns_B4 | LC3 (TP6) | 17.500 | 2.2588e-05 | 8.261 | 0.88373 |
| ns_B4 | LC3 (TP8) | 18.120 | 1.2305e-06 | 9.878 | 0.61908 |
| ns_B4 | LC3 (TP10) | 31.120 | 1.8244e-08 | 10.000 | 0.033471 |
| ns_B4 | LC3 (TP12) | 63.970 | 2.5061e-10 | 10.000 | 0.0009416 |
| ns_B4 | LC3 (TP14) | 14.450 | 0.00097031 | 4.842 | 1.322 |

Table 3.70: Simulation results of Problem ns_B5

| Problems | Methods | c5t | c5w | c6t | c6w |
|----------|----------------|-------|------------|--------|------------|
| ns_B5 | TAYLOR4 (TP4) | 0.340 | 0.00024281 | 10.000 | 0.023851 |
| ns_B5 | TAYLOR4 (TP6) | 0.420 | 4.2469e-06 | 10.000 | 0.00038581 |
| ns_B5 | TAYLOR4 (TP8) | 0.730 | 6.5182e-08 | 10.000 | 0.00039017 |
| ns_B5 | TAYLOR4 (TP10) | 1.880 | 7.267e-10 | 10.000 | 2.2839e-05 |
| ns_B5 | TAYLOR4 (TP12) | 4.040 | 7.7236e-12 | 10.000 | 4.2675e-07 |
| ns_B5 | TAYLOR4 (TP14) | 0.340 | 0.00024281 | 10.000 | 0.023851 |
| ns_B5 | RK4 (TP4) | 0.230 | 0.00012717 | 10.000 | 0.018828 |
| ns_B5 | RK4 (TP6) | 0.280 | 3.3117e-06 | 10.000 | 0.00050419 |
| ns_B5 | RK4 (TP8) | 0.710 | 4.9849e-08 | 10.000 | 0.00028774 |
| ns_B5 | RK4 (TP10) | 0.960 | 5.6878e-10 | 10.000 | 2.2317e-05 |
| ns_B5 | RK4 (TP12) | 2.320 | 5.9515e-12 | 10.000 | 3.4427e-07 |
| ns_B5 | RK4 (TP14) | 0.230 | 0.00012717 | 10.000 | 0.018828 |
| ns_B5 | LA3 (TP4) | 0.230 | 4.6884e-05 | 10.000 | 0.085944 |
| ns_B5 | LA3 (TP6) | 0.250 | 3.5205e-06 | 10.000 | 0.00074212 |
| ns_B5 | LA3 (TP8) | 0.410 | 5.4075e-08 | 10.000 | 0.00049104 |
| ns_B5 | LA3 (TP10) | 0.860 | 6.3172e-10 | 10.000 | 2.4014e-05 |
| ns_B5 | LA3 (TP12) | 1.930 | 6.5794e-12 | 10.000 | 3.4972e-07 |
| ns_B5 | LA3 (TP14) | 0.230 | 4.6884e-05 | 10.000 | 0.085944 |
| ns_B5 | LC3 (TP4) | 0.230 | 4.1633e-05 | 10.000 | 0.099077 |
| ns_B5 | LC3 (TP6) | 0.260 | 3.8362e-06 | 10.000 | 0.0014268 |
| ns_B5 | LC3 (TP8) | 0.390 | 5.3813e-08 | 10.000 | 0.00028027 |
| ns_B5 | LC3 (TP10) | 0.770 | 4.7183e-10 | 10.000 | 2.0975e-05 |
| ns_B5 | LC3 (TP12) | 1.730 | 4.3485e-12 | 10.000 | 2.4349e-07 |
| ns_B5 | LC3 (TP14) | 0.230 | 4.1633e-05 | 10.000 | 0.099077 |

| Problems | Methods | c5t | c5w | c6t | c6w |
|----------|----------------|---------|------------|--------|----------|
| ns_D1 | TAYLOR4 (TP4) | 25.810 | 0.006207 | 5.396 | 2.925 |
| ns_D1 | TAYLOR4 (TP6) | 34.860 | 0.0034041 | 9.153 | 1.8021 |
| ns_D1 | TAYLOR4 (TP8) | 53.420 | 0.0033342 | 10.000 | 1.3536 |
| ns_D1 | TAYLOR4 (TP10) | 99.050 | 0.0033352 | 8.847 | 2.0949 |
| ns_D1 | TAYLOR4 (TP12) | 188.630 | 0.0016658 | 8.083 | 2.7048 |
| ns_D1 | TAYLOR4 (TP14) | 25.820 | 0.006207 | 5.396 | 2.925 |
| ns_D1 | RK4 (TP4) | 20.920 | 0.0028112 | 6.395 | 1.7957 |
| ns_D1 | RK4 (TP6) | 31.130 | 0.0016874 | 10.000 | 0.38124 |
| ns_D1 | RK4 (TP8) | 44.450 | 0.0016637 | 10.000 | 0.30351 |
| ns_D1 | RK4 (TP10) | 46.830 | 0.0016633 | 10.000 | 0.21176 |
| ns_D1 | RK4 (TP12) | 90.540 | 0.00083237 | 9.009 | 1.8083 |
| ns_D1 | RK4 (TP14) | 20.910 | 0.0028112 | 6.395 | 1.7957 |
| ns_D1 | LA3 (TP4) | 18.400 | 0.0022911 | 3.265 | 0.023256 |
| ns_D1 | LA3 (TP6) | 24.620 | 0.00073054 | 9.243 | 1.7605 |
| ns_D1 | LA3 (TP8) | 32.510 | 0.0006961 | 10.000 | 0.5364 |
| ns_D1 | LA3 (TP10) | 48.120 | 0.00069491 | 10.000 | 0.532 |
| ns_D1 | LA3 (TP12) | 91.680 | 0.0006949 | 8.836 | 1.8679 |
| ns_D1 | LA3 (TP14) | 18.470 | 0.0022911 | 3.265 | 0.023256 |
| ns_D1 | LC3 (TP4) | 18.290 | 0.0019492 | 3.291 | 0.023226 |
| ns_D1 | LC3 (TP6) | 24.570 | 0.00026326 | 9.563 | 1.9057 |
| ns_D1 | LC3 (TP8) | 30.500 | 0.00022948 | 10.000 | 0.19742 |
| ns_D1 | LC3 (TP10) | 48.300 | 0.00022838 | 10.000 | 0.72164 |
| ns_D1 | LC3 (TP12) | 95.330 | 9.4802e-05 | 9.079 | 1.8038 |
| ns_D1 | LC3 (TP14) | 18.230 | 0.0019492 | 3.291 | 0.023226 |

Table 3.72: Simulation results of Problem ns_E1

| Problems | Methods | c5t | c5w | c6t | c6w |
|----------|----------------|-------|-----------|--------|------------|
| ns_E1 | TAYLOR4 (TP4) | 0.570 | 1.4351 | 2.719 | 18.977 |
| ns_E1 | TAYLOR4 (TP6) | 0.930 | 0.42517 | 2.000 | 0.00010071 |
| ns_E1 | TAYLOR4 (TP8) | 1.450 | 0.20741 | 2.000 | 8.381e-06 |
| ns_E1 | TAYLOR4 (TP10) | 3.080 | 0.13387 | 2.000 | 1.0032e-07 |
| ns_E1 | TAYLOR4 (TP12) | 7.210 | 0.050696 | 10.000 | 22.567 |
| ns_E1 | TAYLOR4 (TP14) | 0.570 | 1.4351 | 2.719 | 18.977 |
| ns_E1 | RK4 (TP4) | 0.420 | 0.012668 | 10.000 | 1.2214 |
| ns_E1 | RK4 (TP6) | 0.500 | 0.017561 | 2.000 | 6.1085e-05 |
| ns_E1 | RK4 (TP8) | 0.700 | 0.031314 | 2.000 | 5.143e-06 |
| ns_E1 | RK4 (TP10) | 1.140 | 0.030614 | 2.000 | 6.6196e-08 |
| ns_E1 | RK4 (TP12) | 2.540 | 0.031647 | 10.000 | 0.91288 |
| ns_E1 | RK4 (TP14) | 0.410 | 0.012668 | 10.000 | 1.2214 |
| ns_E1 | LA3 (TP4) | 0.410 | 0.013204 | 10.000 | 0.34595 |
| ns_E1 | LA3 (TP6) | 0.500 | 0.010426 | 2.000 | 8.1548e-05 |
| ns_E1 | LA3 (TP8) | 0.690 | 0.013066 | 2.000 | 3.5913e-06 |
| ns_E1 | LA3 (TP10) | 1.110 | 0.015096 | 2.000 | 5.1198e-08 |
| ns_E1 | LA3 (TP12) | 2.420 | 0.011244 | 10.000 | 0.33604 |
| ns_E1 | LA3 (TP14) | 0.420 | 0.013204 | 10.000 | 0.34595 |
| ns_E1 | LC3 (TP4) | 0.410 | 0.0095702 | 10.000 | 0.26912 |
| ns_E1 | LC3 (TP6) | 0.500 | 0.01023 | 2.000 | 8.7855e-05 |
| ns_E1 | LC3 (TP8) | 0.720 | 0.010676 | 2.000 | 3.8404e-06 |
| ns_E1 | LC3 (TP10) | 1.080 | 0.0095686 | 2.000 | 4.2571e-08 |
| ns_E1 | LC3 (TP12) | 2.050 | 0.0091033 | 10.000 | 0.22942 |
| ns_E1 | LC3 (TP14) | 0.410 | 0.0095702 | 10.000 | 0.26912 |

Table 3.73: Simulation results of Problem ns_E2

| Problems | Methods | c5t | c5w | c6t | c6w |
|----------|----------------|-------|------------|--------|------------|
| ns_E2 | TAYLOR4 (TP4) | 0.080 | 0.00040596 | 10.000 | 0.015868 |
| ns_E2 | TAYLOR4 (TP6) | 0.120 | 5.6232e-06 | 2.000 | 0.00010071 |
| ns_E2 | TAYLOR4 (TP8) | 0.180 | 7.7141e-08 | 2.000 | 8.381e-06 |
| ns_E2 | TAYLOR4 (TP10) | 0.410 | 8.2963e-10 | 2.000 | 1.0032e-07 |
| ns_E2 | TAYLOR4 (TP12) | 1.010 | 8.848e-12 | 10.000 | 3.459e-07 |
| ns_E2 | TAYLOR4 (TP14) | 0.080 | 0.00040596 | 10.000 | 0.015868 |
| ns_E2 | RK4 (TP4) | 0.050 | 0.0003214 | 10.000 | 0.015258 |
| ns_E2 | RK4 (TP6) | 0.070 | 7.4223e-06 | 2.000 | 6.1085e-05 |
| ns_E2 | RK4 (TP8) | 0.130 | 1.1435e-07 | 2.000 | 5.143e-06 |
| ns_E2 | RK4 (TP10) | 0.280 | 1.8288e-09 | 2.000 | 6.6196e-08 |
| ns_E2 | RK4 (TP12) | 0.670 | 2.3118e-11 | 10.000 | 5.8802e-07 |
| ns_E2 | RK4 (TP14) | 0.050 | 0.0003214 | 10.000 | 0.015258 |
| ns_E2 | LA3 (TP4) | 0.050 | 0.00025275 | 10.000 | 0.066185 |
| ns_E2 | LA3 (TP6) | 0.060 | 3.8427e-06 | 2.000 | 8.1548e-05 |
| ns_E2 | LA3 (TP8) | 0.100 | 7.7582e-08 | 2.000 | 3.5913e-06 |
| ns_E2 | LA3 (TP10) | 0.230 | 1e-09 | 2.000 | 5.1198e-08 |
| ns_E2 | LA3 (TP12) | 0.550 | 1.4015e-11 | 10.000 | 5.7683e-07 |
| ns_E2 | LA3 (TP14) | 0.050 | 0.00025275 | 10.000 | 0.066185 |
| ns_E2 | LC3 (TP4) | 0.050 | 0.00027974 | 10.000 | 0.10799 |
| ns_E2 | LC3 (TP6) | 0.060 | 3.9986e-06 | 2.000 | 8.7855e-05 |
| ns_E2 | LC3 (TP8) | 0.110 | 7.6055e-08 | 2.000 | 3.8404e-06 |
| ns_E2 | LC3 (TP10) | 0.230 | 7.073e-10 | 2.000 | 4.2571e-08 |
| ns_E2 | LC3 (TP12) | 0.580 | 7.085e-12 | 10.000 | 2.6685e-07 |
| ns_E2 | LC3 (TP14) | 0.060 | 0.00027974 | 10.000 | 0.10799 |

Table 3.74: Simulation results of Problem ns_E3

| Problems | Methods | c5t | c5w | c6t | c6w |
|----------|----------------|-------|------------|--------|------------|
| ns_E3 | TAYLOR4 (TP4) | 0.300 | 0.0001603 | 10.000 | 0.015971 |
| ns_E3 | TAYLOR4 (TP6) | 0.500 | 2.0937e-06 | 2.000 | 9.0741e-06 |
| ns_E3 | TAYLOR4 (TP8) | 0.990 | 3.6313e-08 | 2.000 | 2.1037e-07 |
| ns_E3 | TAYLOR4 (TP10) | 2.430 | 4.1545e-10 | 2.000 | 2.2064e-09 |
| ns_E3 | TAYLOR4 (TP12) | 5.990 | 4.5057e-12 | 10.000 | 2.7482e-08 |
| ns_E3 | TAYLOR4 (TP14) | 0.300 | 0.0001603 | 10.000 | 0.015971 |
| ns_E3 | RK4 (TP4) | 0.230 | 0.00012793 | 10.000 | 0.020574 |
| ns_E3 | RK4 (TP6) | 0.320 | 2.9744e-06 | 2.000 | 1.12e-05 |
| ns_E3 | RK4 (TP8) | 0.570 | 5.1365e-08 | 2.000 | 3.5177e-07 |
| ns_E3 | RK4 (TP10) | 1.270 | 5.4612e-10 | 2.000 | 4.1754e-09 |
| ns_E3 | RK4 (TP12) | 3.160 | 5.5893e-12 | 10.000 | 4.885e-08 |
| ns_E3 | RK4 (TP14) | 0.240 | 0.00012793 | 10.000 | 0.020574 |
| ns_E3 | LA3 (TP4) | 0.190 | 0.00026818 | 10.000 | 0.036244 |
| ns_E3 | LA3 (TP6) | 0.340 | 2.8738e-06 | 2.000 | 1.0633e-05 |
| ns_E3 | LA3 (TP8) | 0.540 | 5.2924e-08 | 2.000 | 2.7291e-07 |
| ns_E3 | LA3 (TP10) | 1.230 | 5.7092e-10 | 2.000 | 2.9675e-09 |
| ns_E3 | LA3 (TP12) | 3.000 | 5.9095e-12 | 10.000 | 3.4149e-08 |
| ns_E3 | LA3 (TP14) | 0.190 | 0.00026818 | 10.000 | 0.036244 |
| ns_E3 | LC3 (TP4) | 0.210 | 0.00014667 | 10.000 | 0.047597 |
| ns_E3 | LC3 (TP6) | 0.300 | 1.9905e-06 | 2.000 | 9.1701e-06 |
| ns_E3 | LC3 (TP8) | 0.560 | 3.2515e-08 | 2.000 | 2.1352e-07 |
| ns_E3 | LC3 (TP10) | 1.300 | 3.5506e-10 | 2.000 | 2.2933e-09 |
| ns_E3 | LC3 (TP12) | 3.230 | 3.6904e-12 | 10.000 | 2.7388e-08 |
| ns_E3 | LC3 (TP14) | 0.210 | 0.00014667 | 10.000 | 0.047597 |

Table 3.75: Simulation results of Problem ns_E4

| Problems | Methods | c5t | c5w | c6t | c6w |
|----------|----------------|-------|------------|--------|------------|
| ns_E4 | TAYLOR4 (TP4) | 0.040 | 1.2137e-09 | 10.000 | 3.702e-06 |
| ns_E4 | TAYLOR4 (TP6) | 0.040 | 1.2137e-09 | 2.000 | 7.75e-08 |
| ns_E4 | TAYLOR4 (TP8) | 0.040 | 1.2137e-09 | 2.000 | 7.75e-08 |
| ns_E4 | TAYLOR4 (TP10) | 0.040 | 9.0002e-10 | 2.000 | 2.5812e-09 |
| ns_E4 | TAYLOR4 (TP12) | 0.060 | 1.4914e-11 | 10.000 | 1.7366e-10 |
| ns_E4 | TAYLOR4 (TP14) | 0.040 | 1.2137e-09 | 10.000 | 3.702e-06 |
| ns_E4 | RK4 (TP4) | 0.020 | 7.5673e-11 | 10.000 | 7.2614e-07 |
| ns_E4 | RK4 (TP6) | 0.020 | 7.5673e-11 | 2.000 | 3.0567e-09 |
| ns_E4 | RK4 (TP8) | 0.020 | 7.5673e-11 | 2.000 | 3.0567e-09 |
| ns_E4 | RK4 (TP10) | 0.020 | 7.5673e-11 | 2.000 | 1.7044e-09 |
| ns_E4 | RK4 (TP12) | 0.030 | 1.1987e-11 | 10.000 | 2.519e-10 |
| ns_E4 | RK4 (TP14) | 0.020 | 7.5673e-11 | 10.000 | 7.2614e-07 |
| ns_E4 | LA3 (TP4) | 0.020 | 2.0709e-10 | 10.000 | 3.5712e-07 |
| ns_E4 | LA3 (TP6) | 0.020 | 2.0709e-10 | 2.000 | 1.1984e-08 |
| ns_E4 | LA3 (TP8) | 0.020 | 2.0709e-10 | 2.000 | 1.1984e-08 |
| ns_E4 | LA3 (TP10) | 0.020 | 2.0709e-10 | 2.000 | 2.0098e-09 |
| ns_E4 | LA3 (TP12) | 0.030 | 2.4286e-11 | 10.000 | 1.1126e-10 |
| ns_E4 | LA3 (TP14) | 0.020 | 2.0709e-10 | 10.000 | 3.5712e-07 |
| ns_E4 | LC3 (TP4) | 0.020 | 8.4192e-11 | 10.000 | 3.6162e-07 |
| ns_E4 | LC3 (TP6) | 0.020 | 8.4192e-11 | 2.000 | 6.1039e-09 |
| ns_E4 | LC3 (TP8) | 0.020 | 8.4192e-11 | 2.000 | 6.1039e-09 |
| ns_E4 | LC3 (TP10) | 0.020 | 8.4192e-11 | 2.000 | 2.083e-09 |
| ns_E4 | LC3 (TP12) | 0.030 | 1.1283e-11 | 10.000 | 1.1097e-10 |
| ns_E4 | LC3 (TP14) | 0.020 | 8.4192e-11 | 10.000 | 3.6162e-07 |

3.2.3 Discussion

3.3 Other problems

some problems very hard and proof that we win ?

3.3.1 Affine-uncertain

LC3 1e-12

Solution at t=2.000000 :

([-2.39334, 2.05433] ; [-2.19454, 2.48459] ; [-0.0626994, 0.0651442] ; [-0.0672006, 0.0606297] ; [-0

Diameter : (4.44767 ; 4.67913 ; 0.127844 ; 0.12783 ; 0.120356)

Rejected picard :2

Accepted picard :673

Step min :0.00174779

Step max :0.00313099

Truncature error max :2.06329e-12

3.3.2 circle

rk4 1e-8

Solution at t=100.000000 :

([0.476077, 0.622885] ; [0.763549, 0.910451])

Diameter : (0.146808 ; 0.146902)

Rejected picard :0

Accepted picard :1400

Step min :0.01

Step max :0.0730046

Truncature error max :1.60132e-08

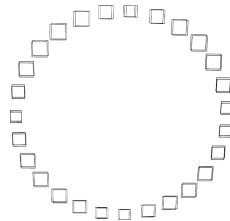


Figure 3.3: Simulation of the circle system

3.3.3 Lambert_prob2_lin_p213

Solution at t=10.000000 :

([-0.544487, -0.543374] ; [-0.839843, -0.838119] ; [10, 10])

Diameter : (0.00111361 ; 0.00172438 ; 0)

Rejected picard :3

Accepted picard :26873

Step min :0.000204486

Step max :0.00040387

Truncature error max :7.99348e-06



Figure 3.4: Simulation of the Lambert linear system

3.3.4 Lambert_stiff_nl_p223

```

Solution at t=1.500000 :
([0.399984, 0.400016] ; [0.00193044, 0.00193047] ; [2.5, 2.5])
Diameter : (3.26431e-05 ; 3.27931e-08 ; 9.19886e-12)
Rejected picard :1
Accepted picard :21060
Step min :4.79902e-06
Step max :0.0152239
Truncature error max :2.70897e-08

```



Figure 3.5: Simulation of the Lambert stiff system

3.3.5 Lorentz

```

Solution at t=4.000000 :
([-4.89107, -4.60361] ; [-0.207979, 0.199418] ; [28.8958, 29.2391])
Diameter : (0.287467 ; 0.407397 ; 0.343226)
Rejected picard :5
Accepted picard :7419
Step min :0.0003125
Step max :0.000924129
Truncature error max :2.52501e-13

```

3.3.6 moins10u

3.3.7 oil-reservoir

```

meth=imidpoint prec=1e-12 picard=taylor3 hmin=1e-6
0.001

```

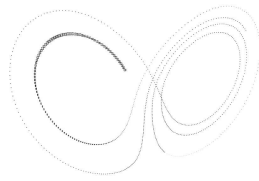


Figure 3.6: Simulation of the Lorenz system

```

Solution at t=50.000000 :
([-8.27752, -8.27751] ; [-0.224547, -0.224547])
Diameter : (6.2308e-06 ; 1.56923e-07)
Rejected picard :3
Accepted picard :71076
Step min :1e-06
Step max :0.016677
Truncature error max :6.57475e-11

```

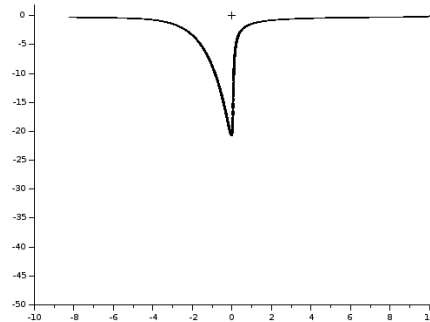


Figure 3.7: Simulation of the oil-reservoir system (stiffness= $1e - 03$)

0.0001

```

Solution at t=50.000000 :
([-8.56149, -8.56146] ; [-0.216578, -0.216577])
Diameter : (2.91622e-05 ; 6.85792e-07)
Rejected picard :2
Accepted picard :73200
Step min :1e-06
Step max :0.0166808
Truncature error max :2.2382e-08

```

3.3.8 vanderpol

```

Solution at t=50.000000 :
([-2.03535, -1.97923] ; [0.0419892, 0.0988844])
Diameter : (0.0561216 ; 0.0568952)
Rejected picard :1

```

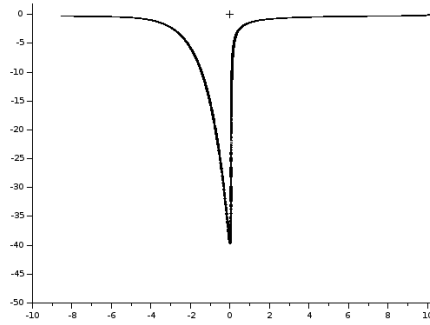


Figure 3.8: Simulation of the oil-reservoir system (stiffness= $1e - 04$)

```
Accepted picard :6789
Step min :0.00140294
Step max :0.012461
Truncature error max :2.53534e-12
```

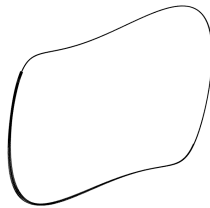


Figure 3.9: Simulation of the vanderpol system ($\mu = 1$)

```
Solution at t=40.000000 :
([1.0493, 1.49018] ; [-0.879307, -0.404504])
Diameter : (0.440879 ; 0.474803)
Rejected picard :2
Accepted picard :7163
Step min :0.00217604
Step max :0.0106292
Truncature error max :3.89696e-12
```

3.3.9 volterra

```
Solution at t=5.488138 :
([1, 1] ; [3, 3])
Diameter : (2.25385e-10 ; 3.90135e-10)
Rejected picard :3
Accepted picard :1642
Step min :0.00115543
Step max :0.00830212
Truncature error max :3.11242e-14
```

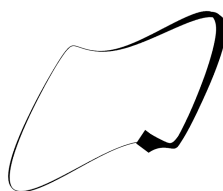


Figure 3.10: Simulation of the vanderpol system ($\mu = 2$)

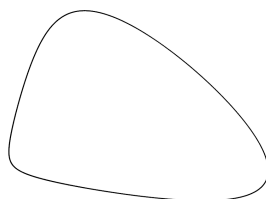


Figure 3.11: Simulation of the volterra system

```

Solution at t=5.488138 :
([0.919632, 1.08037] ; [2.92806, 3.07194])
Diameter : (0.160737 ; 0.14388)
Rejected picard :3
Accepted picard :1702
Step min :0.000912773
Step max :0.00819498
Truncature error max :3.13532e-14

```

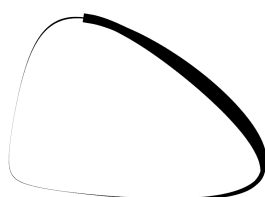


Figure 3.12: Simulation of the volterra system with uncertainties

3.3.10 orbit

Imidpoint

Solution at t=5.000000 :

```
([-0.00635623, 0.0513502] ; [0.826695, 0.905428] ; [-0.188858, -0.0486894] ; [-0.496981, -0.351559])
Diameter : (0.0577065 ; 0.0787324 ; 0.140169 ; 0.145422)
Rejected picard :0
Accepted picard :45489
Step min :1e-07
Step max :0.000310353
Truncature error max :2.18118e-12
```

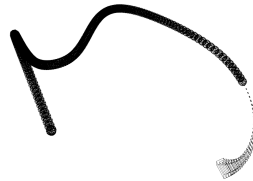


Figure 3.13: Simulation of the orbit system

3.3.11 Neumaier 1

3.3.12 Neumaier 2

3.3.13 Neumaier 3

3.3.14 Quadratic

3.3.15 Rossler

```
Solution at t=50.000000 :
([10.1496, 11.4172] ; [-7.78271, -5.69522] ; [0.0544862, 0.0971181])
Diameter : (1.26763 ; 2.08749 ; 0.0426319)
Rejected picard :6
Accepted picard :7123
Step min :0.001
Step max :0.0132739
Truncature error max :1.3313e-11
```

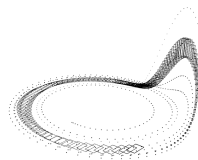


Figure 3.14: Simulation of the Rossler system

3.3.16 Discussion

In this section, we perform some testes on others problem, coming from the literature. It is not to compare with the other tools, but just to prove that we can model and perform come classical problems.

Chapter 4

Conclusion

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