Energy-conserving Modal Synthesis scheme for Vibrations of Thin Plates in Strongly Nonlinear Regime

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ABSTRACT

Large-amplitude geometric nonlinear vibrations of plates are considered. The von Kármán equations are used as model for both unknowns $w(\mathbf{x}, t)$ (transverse displacement) and $F(\mathbf{x}, t)$ (Airy stress function). The key feature is the use of a modal approach for time-integrating the system with a very large number of degrees-of-freedom. The targeted application is the strongly nonlinear vibration of thin plates characterized by a turbulent state with a cascade of energy from the injection to the dissipative scale, involving a huge dimension of the phase space [1, 2]. More specifically, we are interested in the sound synthesis of cymbals and gong-like instruments, that show this wave turbulence regime when strongly beaten with a vigorous strike. In the case of large gong, the striking audible feature characterizing the sound of the instrument is this build-up of energy to the high frequencies which is the signature of the turbulent cascade.

In order to properly simulate such strongly nonlinear regime, as well as being able to use refined damping models with complex frequency dependence, a modal approach is used, as it allows to: (i) compute with accuracy linear and nonlinear model coefficients with a reasonable amount of computation, (ii) easily implement any damping law through the modal damping factors that can be selected at will. For the time integration of such systems suffering from numerical stiffness as well as high-dimensional phase space (the target application is to routniely use a thousand of modes nonlinearly interacting together), a special emphasis is put on the derivation of an ad-hoc, conservative method. Thanks to symmetry properties of the nonlinear coupling coefficients, an energy-conserving scheme especially designed for the modal equations of the von Kármán perfect and imperfect plates is derived, for arbitrary boundary conditions. The scheme is implicit and conditionally stable, provided that the sampling frequency f_S is chosen such that $f_S > \pi f_N$, where f_N is the largest eigenfrequency of the truncation for the transverse modes.

Two simulations results are shown in Fig. 1, where the special cases of a rectangular plate with transverse simply-supported and in-plane movable edges is considered in Fig. 1(a-b), while a circular plate with a free edge is used for Fig. 1(c-d). The plate is impacted at initial time with a raised cosine external force, where the interaction time as well as the amplitude of the force are the two control parameters. In the first case, the case of a gong is approached with a clear build-up of energy in the first second, signature of the turbulent cascade. The plate is 0.6×0.8m with a thickness of 1 mm. The interaction time is 12 ms with an amplitude of 300N. For this simulation 800 transverse modes are used, and the damping law is selected with a complex frequency dependence as $\omega^{0.6}$.

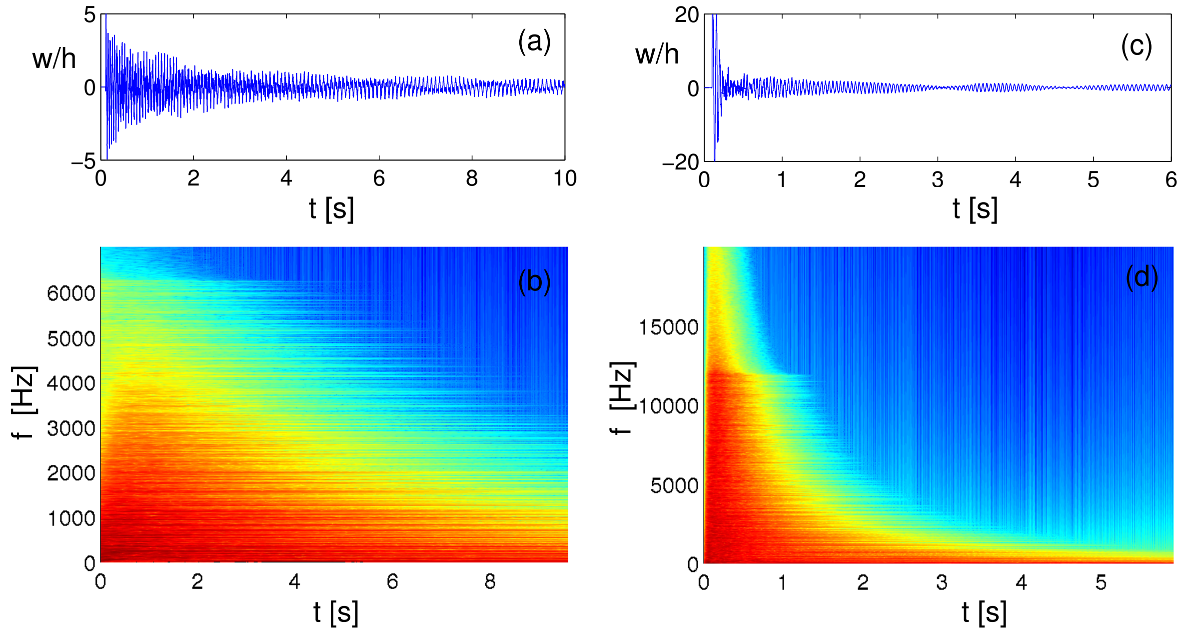

Figure 1: Time series and spectrograms of two simulated strongly nonlinear vibrations. (a)-(b): rectangular plate, (c)-(d): circular plate.

The second case approximates the one of a cymbal with a very short interaction time (4 ms). The radius of the plate is 0.2m, and the thickness 0.5 mm. A large amount of energy is directly transferred up to 15 kHz. 1000 modes have been selected with a sampling rate of 80 kHz.

The results presented in this contribution show undoubtedly the ability of the modal method for the time integration of strongly nonlinear systems, with a huge number of modes interacting together. At the conference, details on the integration scheme will be given, and sound examples will be listened to. The presented method is general, is able to handle very complex nonlinear dynamics for a reasonable amount of computational time coupled with a great accuracy. We believe this should give a new impetus for using a modal approach for nonlinear systems.

References

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