



HAL
open science

DynIBEX: a Differential Constraint Library for Studying Dynamical Systems

Julien Alexandre Dit Sandretto, Alexandre Chapoutot

► **To cite this version:**

Julien Alexandre Dit Sandretto, Alexandre Chapoutot. DynIBEX: a Differential Constraint Library for Studying Dynamical Systems. HSCC, Apr 2016, Vienne, Austria. , HSCC 2016. hal-01302504

HAL Id: hal-01302504

<https://ensta-paris.hal.science/hal-01302504>

Submitted on 14 Apr 2016

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.



DynIBEX: a Differential Constraint Library for Studying Dynamical Systems

Julien Alexandre dit Sandretto and Alexandre Chapoutot

U2IS, ENSTA ParisTech, Université Paris-Saclay, 828 bd des Maréchaux, Palaiseau, France

Contact: alexandre@ensta.fr

A free open-source library combining validated numerical integration methods with a constraint programming approach, as a plugin of IBEX library with:

- validated integration methods based on Runge-Kutta for Ordinary Differential Equations (ODEs) and Differential Algebraic Equations (DAEs)
- operators for satisfaction problems on dynamical systems

<http://perso.ensta-paristech.fr/~chapoutot/dynibex/>

1-Differential Equation Supported

Parametric and Constrained :

$$\begin{cases} \dot{y} = F(t, y, x, p, u) \\ 0 = G(t, y, x, p, u) \\ 0 = H(t, y, p, u) \end{cases}$$

with $y(0) \in [y_0]$, $x(0) \in [x_0]$, $p \in [p]$, $u \in [u]$ and $t \in [0, t_{end}]$.

With:

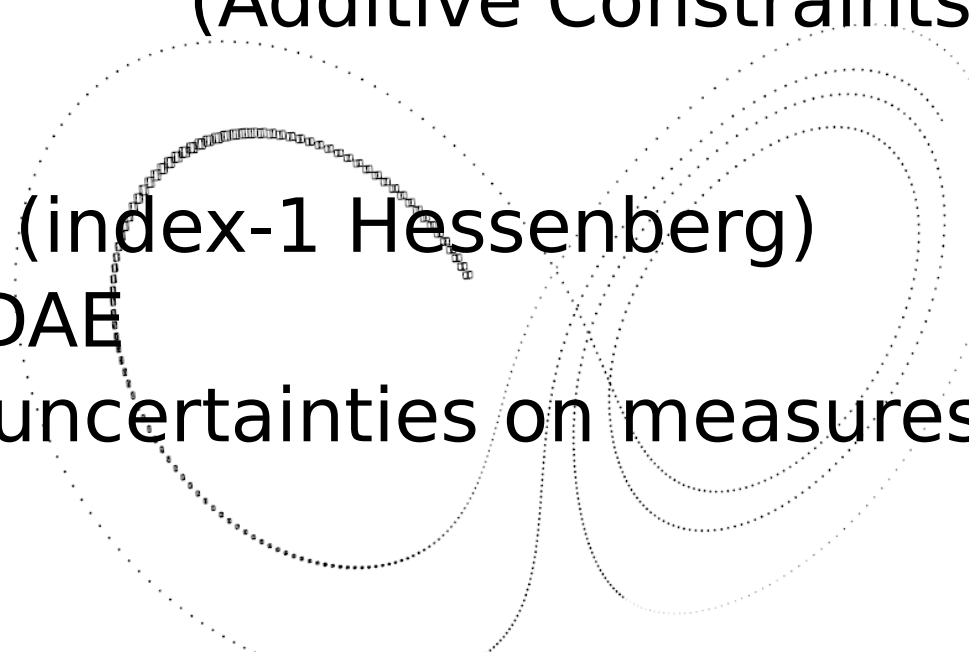
$F : R \times R^n \times R^m \times R^r \times R^s \mapsto R^n$ (Differential Part)

$G : R \times R^n \times R^m \times R^r \times R^s \mapsto R^m$ (DAE part)

$H : R \times R^n \times R^r \times R^s \mapsto R^c$ (Additive Constraints)

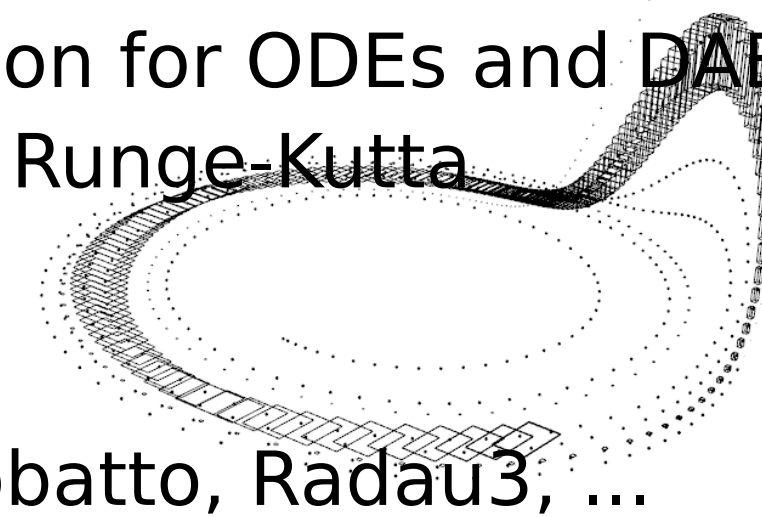
Note that:

- $m=0$: ODE, otherwise DAE (index-1 Hessenberg)
- $c>0$: Constrained ODE or DAE
- $r>0$: Interval Parameters (uncertainties on measures, modeling, parameters)
- $s>0$: Embedded control



3-Validated Simulation with Runge-Kutta

- Proof of existence and uniqueness of solution for ODEs and DAEs
 - Local truncation error computation for any Runge-Kutta method (implicit or explicit)
 - Combined with contractors (HC4)
- => Validated Runge-Kutta: Heun, RK4, Lobatto, Radau3, ...



4- Verification of temporal constraints

Stayed in \mathcal{A} (until τ) : $y(t) \subset \text{Int}(\mathcal{A}), \forall t (\forall t < \tau)$

Included in \mathcal{A} at τ : $y(\tau) \subset \text{Int}(\mathcal{A})$

Has crossed \mathcal{A} (before τ) : $\exists t, y(t) \cap \square \mathcal{A} \neq \emptyset (\exists t < \tau)$

Gone out \mathcal{A} (before τ) : $\exists t, y(t) \cap \square \mathcal{A} = \emptyset (\exists t < \tau)$

Has reached \mathcal{A} : $y(t_{end}) \cap \square \mathcal{A} \neq \emptyset$

Finished in \mathcal{A} : $y(t_{end}) \subset \text{Int}(\mathcal{A})$

5- Example of simulation and constraints

Model of an Hovercraft:

$$\begin{cases} \dot{u} = vr + \frac{1}{m} \left(F_u - \mu_{sec} N f(u) \frac{u}{\sqrt{u^2 + v^2}} - \frac{1}{2} \rho C_d S_u \sqrt{u^2 + v^2} u - D_{11} u \right) \\ \dot{v} = -ur + \frac{1}{m} \left(\frac{1}{4A} C_L S_v F_u \delta - \mu_{sec} N f(v) \frac{v}{\sqrt{u^2 + v^2}} - \frac{1}{2} \rho C_d S_v \sqrt{u^2 + v^2} v - D_{22} v \right) \\ \dot{r} = \frac{1}{I_z} \left(-L \frac{1}{4A} C_L S_y F_u \delta - D_{33} r \right) \\ \dot{x} = \cos(\psi) u - \sin(\psi) v \\ \dot{y} = \sin(\psi) u + \cos(\psi) v \\ \dot{\psi} = r \end{cases}$$

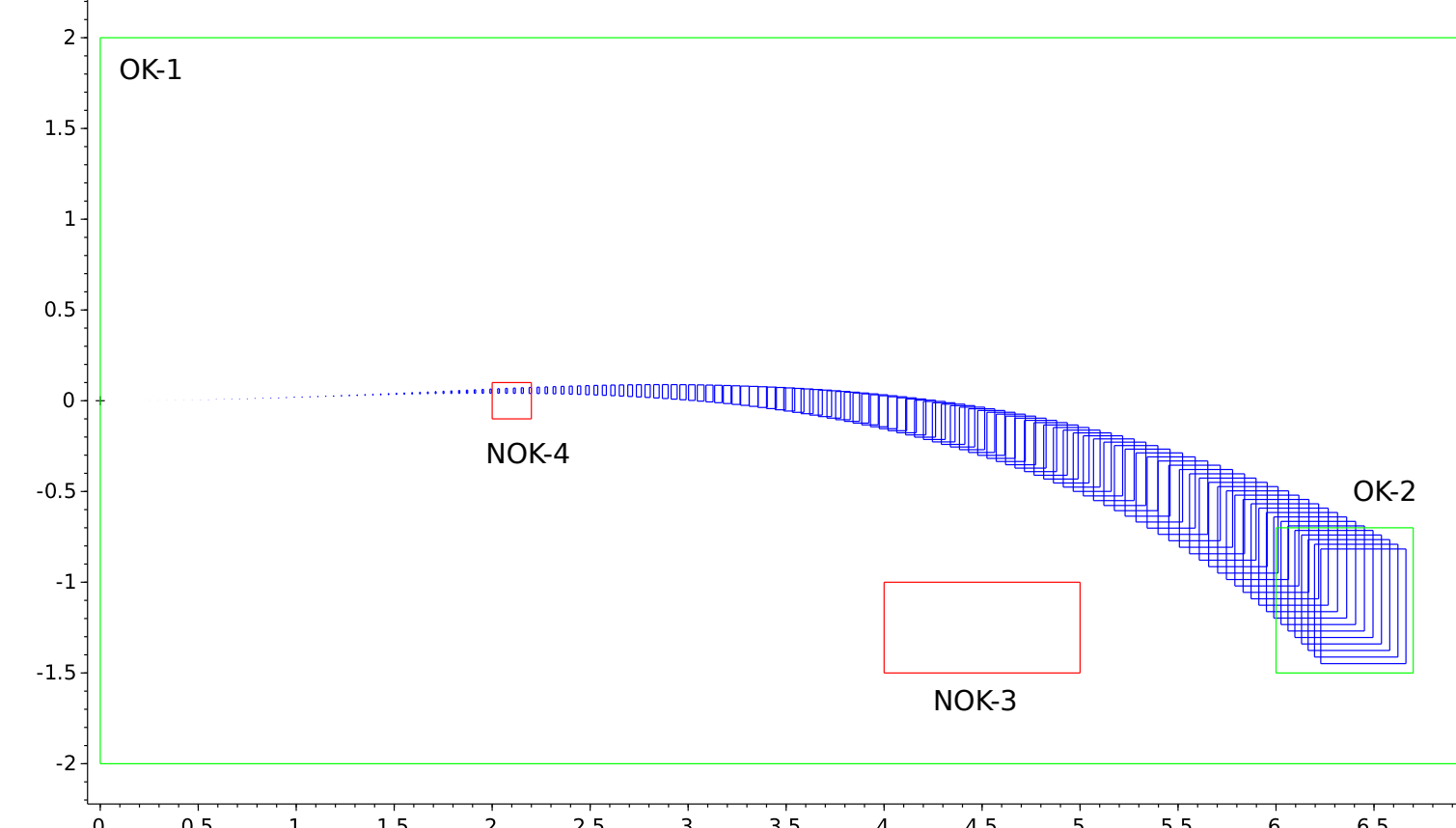
OK-1: Stayed in safe zone

OK-2: Finished in goal

NOK-3: Avoid obstacle

NOK-4: Not in forbidden zone at τ

=> **Design, control synthesis, parameter identification, model simplification, ...**



2-Existing tools in IBEX

A library for interval constraint programming:

- Interval and affine arithmetic
- Function evaluation
- Contractor programming (HC4, Newton, ...)
- Bisection (Branch & Prune)
- Set operators (inclusion, intersection, ...)

DynIBEX applications

Controller synthesis (PID, switched systems), design, viability kernel, predictive control, sensitivity analysis

[1] J.A.D. Sandretto and A. Chapoutot, Validated Explicit and Implicit Runge-Kutta Methods, Reliable Computing, 2016

[2] J.A.D. Sandretto and A. Chapoutot, Validated Simulation of Differential Algebraic Equations, Reliable Computing, 2016

[3] J.A.D. Sandretto, A. Chapoutot, O. Mullier, Tuning PI controller in non-linear uncertain closed-loop systems with interval analysis, SynCoP'2015

[4] D. Monnet, L. Jaulin, J. Ninin, A. Chapoutot, J.A.D. Sandretto, Viability kernel computation based on interval methods, SWIM'2015

[5] A. Le Coënt, J.A.D. Sandretto, A. Chapoutot, L. Fribourg, Control of Nonlinear Switched Systems Based on Validated Simulation, SNR'2016