



**HAL**  
open science

# A phenomenological model for predicting the effect of damping on wave turbulence spectra in vibrating plates

Thomas Humbert, C Josserand, Cyril Touzé, Olivier Cadot

## ► To cite this version:

Thomas Humbert, C Josserand, Cyril Touzé, Olivier Cadot. A phenomenological model for predicting the effect of damping on wave turbulence spectra in vibrating plates. 6th International Conference on Nonlinear Vibrations, Localization and Energy Transfer, Aug 2016, Liège, Belgium. hal-01354764

**HAL Id: hal-01354764**

**<https://ensta-paris.hal.science/hal-01354764>**

Submitted on 19 Aug 2016

**HAL** is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

# A phenomenological model for predicting the effect of damping on wave turbulence spectra in vibrating plates

T. Humbert<sup>1</sup>, C. Josserand<sup>2</sup>, C. Touzé<sup>3</sup> and O. Cadot<sup>3</sup>

<sup>1</sup>SPEC, CNRS, CEA, Université Paris-Saclay  
91191 Gif-sur-Yvette, France, thomas.humbert@cea.fr

<sup>2</sup>Institut D'Alembert, CNRS, UMR 7190  
UPMC, 75005 Paris, France, christophe.josserand@upmc.fr

<sup>3</sup>IMSIA, ENSTA-ParisTech, CNRS, CEA, EDF, Université Paris-Saclay  
91762 Palaiseau, France, cyril.touze@ensta.fr, olivier.cadot@ensta.fr

**Abstract** Thin plates vibrating at large amplitudes may exhibit a strongly nonlinear regime that has to be studied within the framework of wave turbulence. Experimental studies have revealed the importance of the damping on the spectra of wave turbulence, which precludes for a direct comparison with the theoretical results, that assumes a Hamiltonian dynamics. A phenomenological model is here introduced so as to predict the effect of the damping on the turbulence spectra. Self-similar solutions are found and the cut-off frequency is expressed as function of the damping rate and the injected power.

The large amplitude vibrations of thin plates are well described by the Wave Turbulence (WT) theory. The out-of-equilibrium solutions are found from the kinetic equations [1], which admit two different solutions [2]:

- The so-called Rayleigh-Jeans solution where energy is equiparted along all possible lengthscales. In this case the the density of energy  $E^{RJ}(\omega)$  is a constant with respect to the frequency.
- The Kolmogorov-Zacharov (KZ) solution with an energy flux cascading from the largest to the smallest wavelength. In this case the density of energy reads :

$$E^{KZ}(\omega) = A\varepsilon^{\frac{1}{3}} \log^{\frac{1}{3}} \left( \frac{\omega^*}{\omega} \right), \quad (1)$$

with  $A$  a constant,  $\varepsilon$  the (conserved) energy flux and  $\omega^*$  a cut-off frequency.

Numerous experiments have underlined the effect of damping [3,4]. In order to study its effect on the WT spectrum, a phenomenological model is introduced, which describes the temporal and frequency dependence of the energy density  $E(\omega)$ . It reads:

$$\frac{\partial E}{\partial t} = \frac{\partial}{\partial \omega} (\omega E^2 \frac{\partial E}{\partial \omega}) - \hat{\gamma} E, \quad (2)$$

For  $\hat{\gamma} = 0$  (conservative case), the phenomenological model retrieves stationary (RJ and KZ spectra) as well as non-stationary solutions of the kinetic equation [5].

Fig. 1(a) shows the stationary spectra of turbulence obtained from Eq. (2) for a damping law of the form  $\hat{\gamma} = \xi\omega^{0.6}$ , with relative values of  $\xi$  (with respect to the smallest one) ranging from 1 to 5. Interestingly the solutions are self-similar and collapse on a single curve when rescaled by the cut-off frequency  $\omega_c$ , as shown in Fig. 1(b). This solution is different from the KZ spectrum for the conservative case –shown as a green dashed line in Fig. 1(b)– showing undoubtedly the effect of the damping.

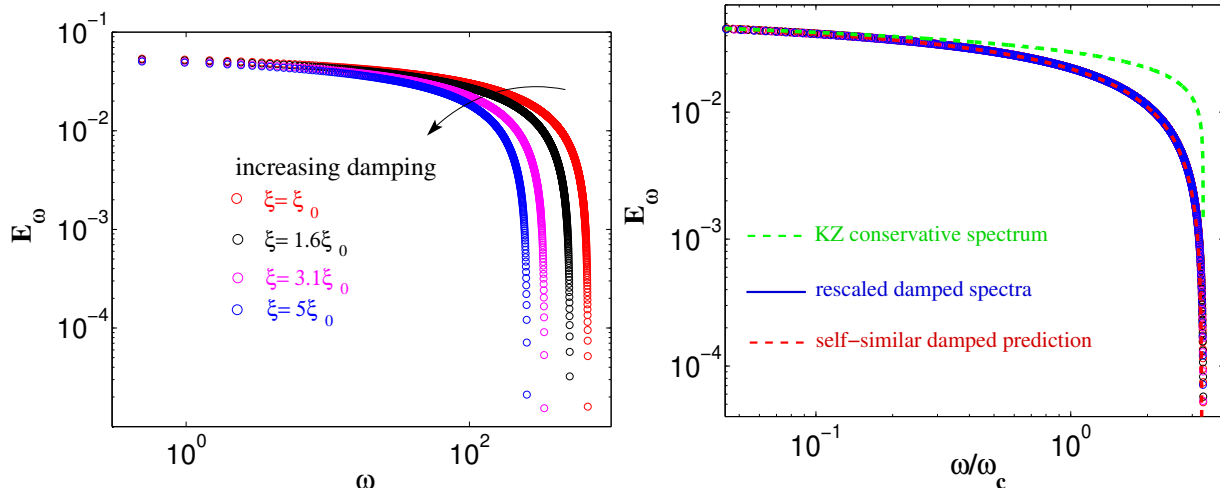


Figure 1: (a) Stationary energy spectrum  $E(\omega)$  for increasing values of the damping. (b) Energy spectra  $E(\omega)$  as functions of the rescaled frequency  $\omega/\omega_c$ .

Let us consider now a general damping rate expressed as  $\hat{\gamma} = \xi\omega^\lambda$ , where the exponent  $\lambda$  controls the frequency dependence of the losses and  $\xi$  its amplitude. The self-similar solution for the spectrum can be expressed as:

$$E(\omega) = \varepsilon_I^{1/3} f_\eta \left( \frac{\omega}{\varepsilon_I^{1/3} \xi z} \right), \quad (3)$$

with  $z = -\frac{1}{1+\lambda}$ ,  $y = \frac{2}{3(1+\lambda)}$ , and  $f_\eta$  solution of the differential equation:

$$\partial_\eta(\eta f_\eta^2 \partial_\eta f_\eta) - f_\eta \eta^\lambda = 0, \quad (4)$$

The solution for this differential equation is shown in Fig. 1(b) as a red dashed line and matches exactly with the direct simulation. Finally one is able to predict the cut-off frequency as function of the damping, and the injected flux as  $\omega_c = \varepsilon_I^{\frac{2}{3(1+\lambda)}} \xi^{-\frac{1}{1+\lambda}}$ , which constitutes the main result of the present study.

## References

- [1] S. Nazarenko, Wave Turbulence, Springer, 2011.
- [2] G. Düring, C. Josserand and S. Rica, Weak turbulence for a vibrating plate: can one hear a Kolmogorov spectrum? Phys. Rev. Lett., 97, 025503, 2006.
- [3] T. Humbert, O. Cadot, G. Düring, C. Josserand, S. Rica and C. Touzé, Wave turbulence in vibrating plates: the effect of damping, Europhys. Lett., 102 (3), 30002, 2013.
- [4] O. Cadot, M. Ducceschi, T. Humbert, B. Miquel, N. Mordant, C. Josserand and C. Touzé, Wave turbulence in vibrating plates, Handbook of Applications of Chaos theory, Chapman and Hall/CRC Press, 2016.
- [5] T. Humbert, C. Josserand, C. Touzé and O. Cadot: Phenomenological model for predicting stationary and non-stationary spectra of wave turbulence in vibrating plates, Physica D, 316, 34-42, 2016.