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Hybrid Cramer Rao Bound on Time Delay Estimation in Rayleigh Fading Channels

Imen Nasr [1,2], , Leïla Najjar Atallah [1], Sofiane Cherif [1] and Benoît Geller [2]

[1]COSIM Research Lab, Higher School of Communications of Tunis (SUP'COM), University of Carthage, Tunisia
[2]U2IS Research Lab, Ecole Nationale Supérieure de Techniques Avancées ENSTA ParisTech, UPSA, France
[1]{nasr.imen, leila.najjar, sofiane.cherif}@supcom.rnu.tn
[2]{benoit.geller}@ensta-paristech.fr

***Abstract*—We propose to derive the analytical expression of the Hybrid Cramer-Rao Bound HCRB for the joint time delay and channel fading estimation. The time delay is considered as a deterministic parameter and the channel is considered as a complex gaussian random variable. The HCRB expression is computed for both the data-aided (DA) and the non data-aided (NDA) modes. The obtained result allows us to assess the performance of on-line and off-line dynamical Rayleigh channel gains estimation. It also evaluates timing recovery techniques under a time varying fading channel.**

I. Introduction

The standard Cramer-Rao Bound (CRB) [1] and the Bayesian Cramer-Rao Bound (BCRB) [2] have been introduced in the literature to find a theoretical minimum mean square error (MSE) limit for respectively, deterministic and stochastic parameter estimation. Closed form expressions of the CRB have been derived in [3] for carrier frequency and phase offset estimation for turbo-coded Square-QAM modulated signals. It was also evaluated in [4] and [5] respectively for SNR and for frequency and phase NDA estimation. For the timing recovery problem, the CRB has been evaluated for DA [6], NDA [7] and with the help of soft information [8], [9] in code-aided (CA) [10], [11] only for a constant delay. The BCRB has been derived for the unknown random phase offset problem [12], [13] in [14]–[16] and for dynamical timing recovery [17], [18].

However, it is frequent that a mixture of deterministic and stochastic parameters has to be jointly estimated. For instance, in real systems, time delay recovery is often associated to dynamic channel estimation. CRB and BCRB are then no more relevant for such estimators designed to jointly recover deterministic and random parameters. By the introduction of a general framework to derive the analytical expression of the BCRB suited for a vector of parameters estimation [2], an extension has been proposed [19] for some practical cases where the vector to be estimated is composed of a mixture of random and deterministic parts. This theoretical bound is called the Hybrid CRB (HCRB).

In [20] a HCRB has been introduced for a dynamical time varying channel, modeled by a random variable, and a deterministic carrier frequency offset estimation in OFDM systems. In [14], [21], this bound has been computed for a dynamical phase offset estimation following a brownian model with a linear unknown drift. The authors of [22] proposed a HCRB for the joint estimation of the pair of dynamical carrier phase-Doppler shift (random part) and the constant time delay (deterministic part). In this proposal, we provide an analytical derivation of the HCRB for a joint estimation of a Rayleigh fading channel and a deterministic time delay.

This paper is organized as follows. In section II, the system model is presented. In section III, closed form expressions of the HCRB for both the DA and the NDA modes are derived. Simulation results are provided in section IV and validate our analysis. The last section concludes our work.

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GLOSSARY OF PRINCIPAL NOTATIONS

- $\Re\{z\}$ and $\Im\{z\}$: the real and imaginary parts of z
- z^*: the conjugate of z
- $\boldsymbol{X}^T$: the transpose of the matrix $\boldsymbol{X}$
- $\dot{f}$ (resp. $\ddot{f}$): the first (resp. the second) derivative of f, for any function f
- $f_X(x)$: the probability density function (pdf) of the random variable X
- $P(X)$: the probability of the random variable X
- $E_y(X)$: the mean value of the random variable X over y
- $|\boldsymbol{X}|$: the determinant of the matrix $\boldsymbol{X}$
- ∇_x: the gradient with respect to x
- $\triangle_x^y = \nabla_x \nabla_y$: the Laplace operator with repect to $y$and x

II. System Model

Let us consider the linearly modulated transmitted signal $s(t)$ such as:

$$s(t) = \sum_i a_i h(t - iT), \tag{1}$$

where a_i denotes the transmitted symbols which are assumed to be statistically independent and equally likely, with normalized energy, $h(t)$ is the impulse response of the root Nyquist transmission filter and T is the symbol period.

Generally, propagation in a radio channel is characterized by a very slowly varying time delay with respect to the channel gains affecting the signal amplitude and phase. Hence, it is reasonable to assume that the time delay remains constant and

the channel gain varies from one measurement to another. The received signal can thus be written as:

$$r(t) = \alpha(t)s(t-\tau) + n(t), \tag{2}$$

where τ is an unknown delay introduced by the channel, $\alpha(t)$ is a complex gaussian process representing the channel fading and $n(t)$ is an additive white Gaussian noise (AWGN) of zero mean and variance σ^2.

The Nyquist pulse $g(t)$ obtained from $h(t)$, with $g(t) = h(t) \otimes h^*(-t)$, satisfies the first Nyquist criterion ($g(nT) = 0$ for any integer $n \neq 0$).

Let us consider the N-dimensional truncated vectors $\mathbf{r} = [r_1, \cdots, r_N]^T$, $\boldsymbol{s(\tau)} = [s_1(\tau), \cdots, s_N(\tau)]^T$, $\boldsymbol{\alpha} = [\alpha_1, \cdots, \alpha_N]^T$ and $\boldsymbol{n} = [n_1, \cdots, n_N]^T$, representing the sampled version of $r(t)$, $s(t-\tau)$ and $n(t)$, respectively taken at the rate F/T. $\boldsymbol{\alpha}$ and τ are assumed to be unknown, respectively random and deterministic parameters. They can be gathered in the following unknown parameter vector:

$$\boldsymbol{\mu} = [\boldsymbol{\alpha}^T, \tau]^T. \tag{3}$$

Let $\hat{\boldsymbol{\mu}}(\boldsymbol{r})$ be an unbiased estimator of the hybrid parameter $\boldsymbol{\mu} = [\boldsymbol{\alpha}, \tau]^T$ based on the observations $\mathbf{r}$. The HCRB can be derived in an on-line context where the estimate of $\boldsymbol{\mu}$ only depends on the current and the previous observations or in an off-line context when the whole observation block is received. This bound is defined as [23]:

$$E_{\mathbf{r},\boldsymbol{\alpha}|\tau}\left[(\hat{\boldsymbol{\mu}}(\boldsymbol{r}) - \boldsymbol{\mu})(\hat{\boldsymbol{\mu}}(\boldsymbol{r}) - \boldsymbol{\mu})^H\right] \geq \boldsymbol{HCRB(\mu)}, \tag{4}$$

where $\mathbf{A} \geq \boldsymbol{B}$ means that $\mathbf{A} - \boldsymbol{B}$ is a positive semi definite matrix.

III. HCRB ANALYTICAL DERIVATION

From [19] and [24], the HCRB can be decomposed into four block sub-matrices:

$$\boldsymbol{HCRB(\mu)} = \begin{pmatrix} \boldsymbol{HCRB}^{11}(\boldsymbol{\alpha}) & \boldsymbol{hcrb}^{12}(\boldsymbol{\alpha},\tau) \\ \boldsymbol{hcrb}^{21}(\boldsymbol{\alpha},\tau) & \text{hcrb}^{22}(\tau) \end{pmatrix}. \tag{5}$$

The HCRB is the inverse of the following hybrid information matrix (HIM) [19]:

$$\boldsymbol{H(\mu)} = E_{\boldsymbol{\alpha}}\left[\boldsymbol{F(\alpha,\tau)}\right] + E_{\boldsymbol{\alpha}|\tau}\left[-\triangle_{\boldsymbol{\mu}}^{\boldsymbol{\mu}} \log\left(P(\boldsymbol{\alpha}|\tau)\right)\right], \tag{6}$$

where $P(\boldsymbol{\alpha}|\tau)$ is the *a priori* knowledge on $\boldsymbol{\alpha}$ and $\boldsymbol{F(\alpha,\tau)}$ is the Fisher information matrix (FIM) defined as:

$$\boldsymbol{F(\alpha,\tau)} = E_{\mathbf{r}|\boldsymbol{\alpha},\tau}\left[-\triangle_{\boldsymbol{\mu}}^{\boldsymbol{\mu}} \log\left(P(\mathbf{r}|\boldsymbol{\alpha},\tau)\right)\right]. \tag{7}$$

$P(\mathbf{r}|\boldsymbol{\alpha},\tau)$ is the conditional pdf of $\mathbf{r}$ given $\boldsymbol{\alpha}$ and τ.

Since $\boldsymbol{\alpha}$ and τ are independent parameters, then $P(\boldsymbol{\alpha}|\tau) = P(\boldsymbol{\alpha})$ and thus:

$$E_{\boldsymbol{\alpha}|\tau}\left[-\triangle_{\boldsymbol{\mu}}^{\boldsymbol{\mu}} \log\left(P(\boldsymbol{\alpha}|\tau)\right)\right] = \begin{pmatrix} E_{\boldsymbol{\alpha}}\left[-\triangle_{\boldsymbol{\alpha}}^{\boldsymbol{\alpha}} \log\left(P(\boldsymbol{\alpha})\right)\right] & \mathbf{0_{N,1}} \\ \mathbf{0_{1,N}} & 0 \end{pmatrix}. \tag{8}$$

The channel fading follows a Rayleigh distribution, therefore, $\boldsymbol{\alpha}$ is a complex gaussian random variable of zero mean and covariance matrix $\boldsymbol{R_\alpha}$. The *a priori* information on $\boldsymbol{\alpha}$ is then given by:

$$P(\boldsymbol{\alpha}) = \frac{1}{|\pi \boldsymbol{R_\alpha}|} \exp\left(-\boldsymbol{\alpha}^H {\boldsymbol{R_\alpha}}^{-1} \boldsymbol{\alpha}\right). \tag{9}$$

By evaluating the second derivative of $\log\left(P(\boldsymbol{\alpha})\right)$ with respect to $\boldsymbol{\alpha}$ and making the expectation, we obtain:

$$E_{\boldsymbol{\alpha}}\left[-\triangle_{\boldsymbol{\alpha}}^{\boldsymbol{\alpha}} \log\left(P(\boldsymbol{\alpha})\right)\right] = {\boldsymbol{R_\alpha}}^{-1}. \tag{10}$$

The covariance matrix can be computed using the following expression [25]:

$$[\boldsymbol{R_\alpha}]_{n,m} = \sigma_\alpha^2 J_0\left(2\pi f_d T(n-m)\right) \tag{11}$$

where σ_α is the standard deviation of the channel fading, f_d is the channel Doppler frequency and $J_0(.)$ is the Bessel function of the first kind.

The FIM can be decomposed into the following block matrices

$$\boldsymbol{F(\alpha,\tau)} = \begin{pmatrix} \boldsymbol{F}^{11}(\boldsymbol{\alpha}) & \boldsymbol{f}^{12}(\boldsymbol{\alpha},\tau) \\ \boldsymbol{f}^{21}(\boldsymbol{\alpha},\tau) & \text{f}^{22}(\tau) \end{pmatrix}, \tag{12}$$

where:

$$\begin{aligned} \boldsymbol{F}^{11}(\boldsymbol{\alpha}) &= E_{\mathbf{r}|\boldsymbol{\alpha},\tau}\left[-\triangle_{\boldsymbol{\alpha}}^{\boldsymbol{\alpha}} \log\left(P(\mathbf{r}|\boldsymbol{\alpha},\tau)\right)\right], & (13)\\ \boldsymbol{f}^{12}(\boldsymbol{\alpha},\tau) &= E_{\mathbf{r}|\boldsymbol{\alpha},\tau}\left[-\triangle_{\boldsymbol{\alpha}}^{\tau} \log\left(P(\mathbf{r}|\boldsymbol{\alpha},\tau)\right)\right], & (14)\\ \boldsymbol{f}^{21}(\boldsymbol{\alpha},\tau) &= E_{\mathbf{r}|\boldsymbol{\alpha},\tau}\left[-\triangle_{\tau}^{\boldsymbol{\alpha}} \log\left(P(\mathbf{r}|\boldsymbol{\alpha},\tau)\right)\right], & (15)\\ \text{f}^{22}(\tau) &= E_{\mathbf{r}|\boldsymbol{\alpha},\tau}\left[-\triangle_{\tau}^{\tau} \log\left(P(\mathbf{r}|\boldsymbol{\alpha},\tau)\right)\right]. & (16) \end{aligned}$$

In order to obtain the FIM, we need to first derive the likelihood of $\mathbf{r}$ expressed as, $P(\mathbf{r}|\boldsymbol{\alpha},\tau)$.

A. Observation Likelihood Function

Based on the whiteness of noise samples, the likelihood function of $\mathbf{r}$ given $\boldsymbol{\alpha}$ and τ is:

$$P(\mathbf{r}|\boldsymbol{\alpha},\tau) = \prod_{k=1}^{N} P(r_k|\alpha_k,\tau), \tag{17}$$

where:

$$P(r_k|\alpha_k,\tau) = \frac{1}{2\pi\sigma^2} \exp\left(-\frac{|r_k - \alpha_k s_k(\tau)|^2}{2\sigma^2}\right). \tag{18}$$

Then (17) leads to:

$$P(\mathbf{r}|\boldsymbol{\alpha},\tau) = \left(\frac{1}{2\pi\sigma^2}\right)^N \exp\left(-\frac{\sum_{k=1}^{N} |r_k - \alpha_k s_k(\tau)|^2}{2\sigma^2}\right). \tag{19}$$

Making N tend to infinity, the observation likelihood function can be written in the continuous format:

$$\begin{aligned} P(\mathbf{r}|\boldsymbol{\alpha},\tau) &= \frac{1}{2\pi\sigma^2} \exp\left(-\frac{\int_{T_0} |r(t) - \alpha(t)s(t-\tau)|^2 \, dt}{2\sigma^2}\right) \\ &= \frac{1}{2\pi\sigma^2} \exp\left(-\frac{\int_{T_0} |r(t)|^2 + |\alpha(t)s(t-\tau)|^2 \, dt}{2\sigma^2} \right. \\ &\quad \left. + \frac{\int_{T_0} \Re\left\{\alpha(t)^* s(t-\tau)^* r(t)\right\} dt}{\sigma^2}\right), \end{aligned} \tag{20}$$

where T_0 is the observation period. We note that $|r(t)|^2$ does not depend on τ. According to [26], [27], when T_0 is large enough, the impact of the time delay on the integral of the second squared term in (20) can be neglected and thus:

$$\int_{T_0} |\alpha(t)s(t-\tau)|^2 \, dt \simeq \int_{T_0} |\alpha(t)s(t)|^2 \, dt = \sum_{k=1}^{N} |\alpha_k|^2 |a_k|^2. \tag{21}$$

Considering:

$$x_k(u) = y_k(u) + v_k(u), \quad (22)$$

$$y_k(u) = \alpha_k \sum_i a_i g\left((k-i)T - (\tau - u)\right), \quad (23)$$

$$v_k(u) = \int_{T_0} h^*(t - kT - u)n(t)dt, \quad (24)$$

then after some mathematical derivations, the observation likelihood becomes:

$$P(\mathbf{r}|\boldsymbol{\alpha}, \tau) = C \exp\left(-\frac{\sum_{k=1}^{N} |\alpha_k a_k|^2 - 2\sum_{k=1}^{N} \Re\left\{\alpha_k^* a_k^* x_k(\tau)\right\}}{2\sigma^2}\right), \quad (25)$$

where C is a constant term with respect to τ. $x_k(\tau)$ is seen as the matched filter output of the received signal taken at instant $kT + \tau$ (time delay perfectly recovered) , $y_k(\tau)$ is the useful part and $v_k(\tau)$ is a colored gaussian noise of zero mean and variance σ^2.

The symbols a_k which appear in the updating term can be known if some pilot symbols are sent within the data frame. In this case, the TED is operating in a DA mode. In order to enhance the spectral efficiency, these pilot symbols can be limited to a short preamble or even omitted and a_k are then estimated at the receiver. The TED is then operating in a Decision Directed (DD) mode. To do so, we usually implement a NDA mode in which a hard estimation of the symbols a_k is used. Nevertheless, this technique performance degrades rapidly at low SNR, where modern systems are constrained to work.

Since we obtained the expression of the observation likelihood we can now compute the FIM following (13)-(16).

B. Derivation of the Fisher information matrix in the DA mode

Based on (13)-(16) and (25), we get:

$$\boldsymbol{F}^{11}(\boldsymbol{\alpha}) = diag(J_1, \cdots, J_N), \quad (26)$$

where:

$$J_k = \frac{|a_k|^2}{\sigma^2}, \quad (27)$$

$$\mathrm{f}^{22}(\tau) = \rho \ddot{g}(0), \quad (28)$$

and $\rho = \frac{\sum_{k=1}^{N} |\alpha_k|^2 |a_k|^2}{\sigma^2}$ is the received SNR. We also obtain $\boldsymbol{f}^{12}(\boldsymbol{\alpha}, \tau) = \boldsymbol{f}^{21}(\boldsymbol{\alpha}, \tau)^T = \mathbf{0_{N,1}}$. This is due to the fact that $\boldsymbol{\alpha}$ and τ are two independent parameters.

In order to compute the analytical expression of the HIM given by (6), we further need to average the obtained FIM components over the vector $\boldsymbol{\alpha}$. Given that only $\mathrm{f}^{22}(\tau)$ depends on $\boldsymbol{\alpha}$, then only the FIM related to the time delay τ has to be averaged over $\boldsymbol{\alpha}$.

Averaging (28) over $\boldsymbol{\alpha}$ is equivalent to averaging it over the SNR. We finally obtain:

$$E_{\boldsymbol{\alpha}}\left[\mathrm{f}^{22}(\tau)\right] = \bar{\rho}\ddot{g}(0), \quad (29)$$

where $\bar{\rho}$ is the SNR averaged over the fading channel.

C. Derivation of the Fisher information matrix in the NDA mode

In this paragraph, we provide the expression of the FIM components in the NDA mode for BPSK and QPSK modulated signals. In comparison with the DA mode, only the expression of $\mathrm{f}^{22}(\tau)$ changes. For BPSK and QPSK signals, $|a_k|^2 = 1$ in (27). The derivation of $\mathrm{f}^{22}(\tau)$ for BPSK and QPSK modulations when the transmitted symbols are unknown at the receiver are given in [7] for an AWGN channel by:

$$\mathrm{f}^{22}(\tau) = 4\rho\Big[\left(1 - \frac{1}{\sqrt{2\pi}} e^{-\rho}\beta(\rho)\right)\left(\rho \sum_{m=1}^{N}\sum_{n=1}^{N} \dot{g}((m-n)T) - \frac{N}{2}\ddot{g}(0)\right) - \rho \sum_{m=1}^{N}\sum_{n=1}^{N} \dot{g}((m-n)T)\Big], \quad (30)$$

for a BPSK modulation where:

$$\beta(\rho) = \int_{-\infty}^{+\infty} \frac{e^{-\frac{x^2}{2}}}{\cosh\left(\sqrt{2\rho}x\right)} dx, \quad (31)$$

and:

$$\mathrm{f}^{22}(\tau) = \left(2\rho^2 \sum_{m=1}^{N}\sum_{n=1}^{N} \dot{g}^2((m-n)T) - 2N\rho\ddot{g}(0)\right) \times \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} \frac{g_\rho^2(x)}{G_\rho(x)} e^{-\frac{x^2}{2}} dx - \frac{\rho}{\pi}\left(\int_{-\infty}^{+\infty} x g_\rho(x)\right)^2 e^{-\frac{x^2}{2}} dx \times \sum_{m=1}^{N}\sum_{n=1}^{N} \dot{g}^2((m-n)T), \quad (32)$$

for a QPSK modulation where:

$$g_\rho(x) = 2e^{-2\rho} \sinh\left(2\sqrt{\rho}x\right), \quad (33)$$

and

$$G_\rho(x) = 2e^{-2\rho} \cosh\left(2\sqrt{\rho}x\right). \quad (34)$$

Having a Rayleigh fading channel, then the SNR ρ is exponentially distributed and the pdf of ρ is given by:

$$P(\rho) = \frac{1}{\bar{\rho}} \exp\left(-\frac{\rho}{\bar{\rho}}\right), \text{ for } \rho \geq 0. \quad (35)$$

In order to obtain the final expression of the HIM, we need to average the analytical expressions of $\mathrm{f}^{22}(\tau)$ for BPSK and QPSK modulations over the pdf of ρ for $\rho \geq 0$. Since it is hard to obtain analytically a closed form expression of the integrals given by (30) and (32) and to make the averaging of the obtained result over the SNR distribution, a numerical integration can be used to obtain $E_{\boldsymbol{\alpha}}\left[\mathrm{f}^{22}(\tau)\right]$. We note that the integrals in (30) and (32) decrease rapidly in terms of x. Thus, the integrand functions can be approximated by a finite Riemann integration over an interval $[-D, +D]$ instead of $]-\infty, +\infty[$ with an integration step δ. The same technique can be used to make the averaging over the SNR on the interval $[0, D]$ instead of $[0, +\infty[$.

D. Inversion of the HIM

Based on (6) the HIM is given by:

$$\boldsymbol{H}(\boldsymbol{\mu}) = \begin{pmatrix} E_{\boldsymbol{\alpha}}\left(\boldsymbol{F}^{11}(\boldsymbol{\alpha})\right) + \boldsymbol{R_\alpha}^{-1} & \mathbf{0_{N,1}} \\ \mathbf{0_{1,N}} & E_{\boldsymbol{\alpha}}\left[\mathrm{f}^{22}(\tau)\right] \end{pmatrix} \quad (36)$$

We note that $\boldsymbol{H}(\boldsymbol{\mu})$ is a block diagonal matrix thus using the block matrices inversion formula [1], the HCRB is then given by:

$$\boldsymbol{HCRB}(\boldsymbol{\mu}) = \begin{pmatrix} \left(E_{\boldsymbol{\alpha}}\left(\boldsymbol{F}^{11}(\boldsymbol{\alpha})\right) + \boldsymbol{R_\alpha}^{-1}\right)^{-1} & \mathbf{0_{N,1}} \\ \mathbf{0_{1,N}} & \left(E_{\boldsymbol{\alpha}}\left[\mathrm{f}^{22}(\tau)\right]\right)^{-1} \end{pmatrix} = \begin{pmatrix} \boldsymbol{HCRB_\alpha} & \mathbf{0_{N,1}} \\ \mathbf{0_{1,N}} & \mathrm{hcrb}_\tau \end{pmatrix} \quad (37)$$

IV. Simulation Results

In this section, we show simulation results of the HCRB for the channel coefficients and the time delay estimation. The diagonal elements of the inverse of $\boldsymbol{HCRB(\mu)}$ give the HCRB with respect to $\boldsymbol{\alpha}$ and τ. The results are provided for BPSK and QPSK modulated signals passed through a root raised cosine filter with a roll-off coefficient equal to 0.3. The up-sampling factor is equal to 8.

Figure 1 shows the HCRBs related to the channel and the time delay estimation versus the observation block length. $\boldsymbol{HCRB_{\alpha}}$ is given in an on-line context, where only the current and the previous samples are taken into account in the channel estimation problem. hcrb_{τ} is computed for both DA and NDA modes and for BPSK and QPSK modulated signals. We have also fixed $f_dT = 0.001$, which means that the channel varies slowly within the observation block. The on-line $\boldsymbol{HCRB_{\alpha}}$ is the same for the different modulations and the transmission modes since it only depends on the channel correlation matrix and the observation noise. We note that, like could be expected, the DA outperforms the NDA mode and that the BPSK modulation leads to lower time synchronization errors than the QPSK modulation.

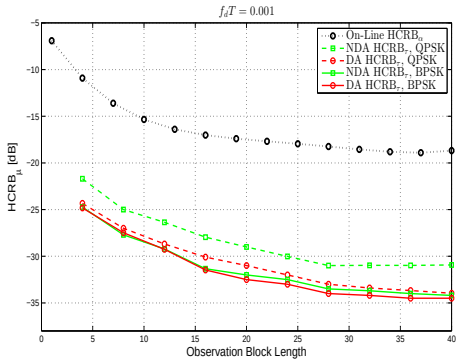


Fig. 1: hcrb_{τ} and $\boldsymbol{HCRB_{\alpha}}$ versus Observation block length (N), $f_dT = 0.001$, SNR=5dB.

In Figure 2, we evaluate the on-line and the off-line $HCRB_{\boldsymbol{\alpha}}$ for various observation block lengths and for $f_dT = 0.001$. Figure 3 displays, for a SNR$=$ 5dB, the on-line and the off-line $HCRB_{\boldsymbol{\alpha}}$ for $f_dT = 0.1$ (fast time varying channel) and $f_dT = 0.001$ (almost invariant channel). It is shown that we can reach better estimation performance with an off-line technique, since we can use the whole observation block. The performances at the center of the symbols block are better than that of the borders for an off-line technique because the current, past and future observations are used at the same time for the channel gain estimation. However, at the borders we can only use the current and, according to the end or the beginning of the block, either the past or the future observation. It is also shown that, at the beginning, the on-line bound decreases, therefore the estimation performance is improved since the estimator uses more and more observations. After the reception of some signal samples, the bound converges to an asymptote and this is due to the observation noise. For instance, the asymptote is reached faster for $f_dT = 0.1$ than for $f_dT = 0.001$.

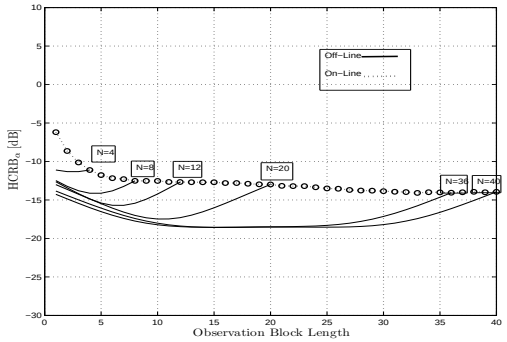


Fig. 2: On-line and Off-line $\boldsymbol{HCRB_{\alpha}}$ versus Observation block length (N), SNR=5dB.

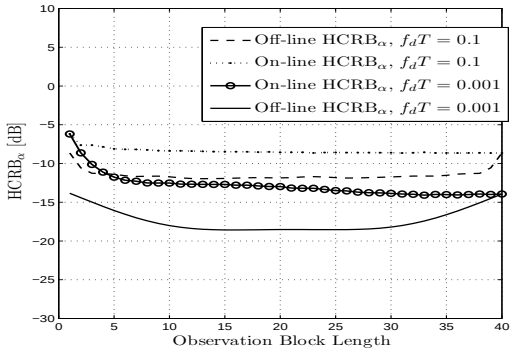


Fig. 3: On-line and Off-line $\boldsymbol{HCRB_{\alpha}}$ versus Observation block length (N), SNR=5dB.

In Figure 4, we display the evolution of the $\boldsymbol{HCRB_{\alpha}}$ with the SNR in the on-line and off-line scenarios and for different Doppler frequency values. It is shown that $\boldsymbol{HCRB_{\alpha}}$ decreases with the SNR and increases with fast varying channels.

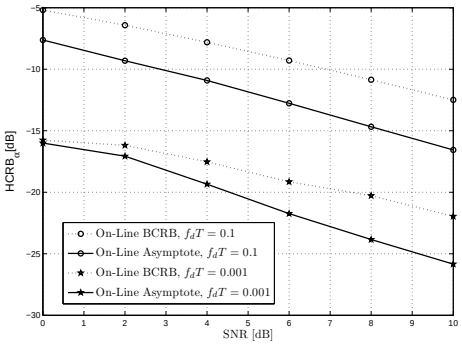


Fig. 4: On-line and Off-line $\boldsymbol{HCRB_{\alpha}}$ versus SNR, $N = 40$.

In Figures 5 and 6, we evaluate, for respectively BPSK and QPSK modulations, the evolution of hcrb_{τ} with the SNR for $f_dT = 0.1$ and $f_dT = 0.001$ and for various transmission modes (DA and NDA). The number of samples received within

the signal block is equal to 40. As it could be expected, hcrb_τ decreases as a function of SNR. Similar results to the previous cases are obtained by changing the SNR values.

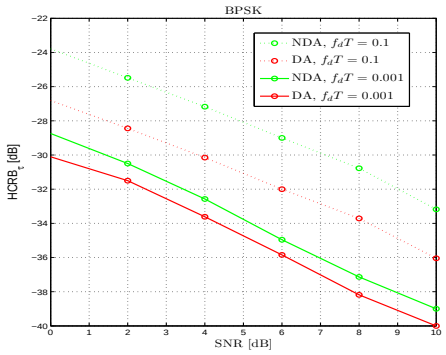


Fig. 5: hcrb_τ versus SNR, Block length= 40.

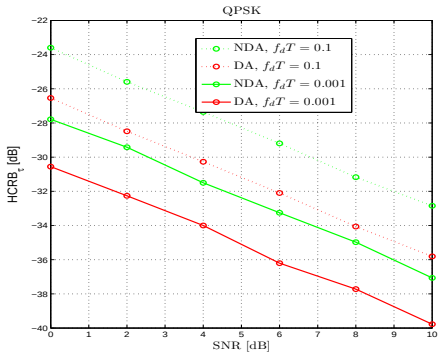


Fig. 6: hcrb_τ versus SNR, Block length= 40.

V. Conclusion

In this contribution, we derive the analytical expression of the HCRB for time delay recovery under a random Rayleigh fading channel in the DA and the NDA modes. Simulation results have shown that we can reach better channel estimation using an off-line technique, where all the observation block is used. It is also shown that we can achieve a better time synchronization performance using the DA mode and with a slowly time varying channel.

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