Model order reduction of nonlinear piezoelectric microstructures through direct parametrisation of invariant manifolds

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Summary. Piezoelectric actuation represents the most effective out-of-plane actuation mechanism for resonant microstructures as scanning micromirrors and piezoelectric ultrasonic transducers. Nevertheless, predicting the dynamic response of such devices at their design stage is often impractical since numerical computation of periodic orbits from finite element systems is computational demanding. In this work, we propose a model order reduction strategy based on the direct parametrisation for invariant manifolds tailored for nonlinear piezoelectric structures. The innovative aspect of the method is the introduction of nonlinear terms that arise due to piezoelectric coupling in the reduction procedure.

Introduction

Among the actuation techniques available for the development of micro-electro-mechanical systems (MEMS), piezoelectric actuation is the most effective to achieve large out-of-plane displacements in resonant actuators as scanning micromirrors and piezoelectric ultrasonic transducers. Indeed, its enhanced linearity, fast feedback response, and high exerted forces provide a mean to achieve high performance with moderate power consumption and easy control of the device. However, the large amplitudes developed by devices as scanning micromirrors is still affected by geometric nonlinearities and by the hysteretic behaviour of piezoelectric materials, which in turn makes the dynamic response of the device nonlinear. Such effects need to be predicted at the design stage of MEMS components, hence making numerical methods of paramount importance for their design. Since full-order numerical simulations are too computational demanding for performing parametrised analyses of mechanical components, dimensionality reduction techniques are essential [1, 2]. Within the context of vibratory systems, the Direct Parametrisation for Invariant Manifolds (DPIM) [3, 4] represents the most effective method for deriving accurate reduced models of structures actuated at resonance. In the present work, we derive an extension of the method initially introduced for systems subjected to geometric nonlinearities to account for the nonlinear converse piezoelectric effect as modeled by the Landau-Devonshire theory of ferroelectrics. The proposed approach is applied to predict the nonlinear dynamic response of MEMS micromirrors and its results are compared with experimental data.

Method

The dynamic response of piezo-MEMS undergoing large displacements and subjected to converse piezoelectric effect is governed by the conservation of linear momentum formulated as follows [5]:

$$\int_\Omega \rho\, \ddot{\mathbf{u}} \cdot \mathbf{w}\, d\Omega + \int_\Omega (\mathbf{S}^e - \mathbf{S}^p) : \delta\mathbf{e}\, d\Omega = 0, \tag{1}$$

where all quantities are reported in material configuration Ω. ρ is the density, $\mathbf{w}$ is the test function, $\delta\mathbf{e}$ is the first variation of the Green-Lagrange strain tensor. The second Piola-Kirchhoff stress tensor is decomposed in elastic $\mathbf{S}^e$ and inelastic components $\mathbf{S}^p$, which in the present framework are given by the following constitutive models [6]:

$$\mathbf{S}^e = \mathcal{A} : \mathbf{e}, \quad \mathbf{S}^p = \mathcal{Q} : \mathcal{A} : (\mathbf{p} \otimes \mathbf{p}). \tag{2}$$

with $\mathcal{A}$ elasticity tensor, $\mathcal{Q}$ electrostrictive tensor, and $\mathbf{p}$ polarisation vector. In general $\mathbf{p}$ is time-dependent and with non-zero average value. Upon finite element discretisation of Eq. (1) and addition of damping the following discrete system of ordinary differential equations is retrieved:

$$\mathbf{M}\ddot{\mathbf{U}} + \mathbf{C}\dot{\mathbf{U}} + \mathbf{K}\mathbf{U} + \mathbf{G}(\mathbf{U}, \mathbf{U}) + \mathbf{H}(\mathbf{U}, \mathbf{U}, \mathbf{U}) = \mathbf{F}^p(\mathbf{U}, \mathbf{p}). \tag{3}$$

with $\mathbf{p}$ function of time. Dimensionality reduction of Eq. (3) is achieved by parametrising the motion of the system along an invariant-based span of the phase space. We recast Eq. (3) in first order by introducing the velocity $\mathbf{V} = \dot{\mathbf{U}}$ and we introduce a nonlinear coordinate change between the normal coordinates $\mathbf{z}$ that describe the motion of the system along the embedding and the nodal degrees of freedom of the model [7, 8]:

$$\mathbf{U} = \mathbf{\Psi}(\mathbf{z}), \quad \mathbf{V} = \mathbf{\Upsilon}(\mathbf{z}). \tag{4}$$

The dynamics along the defined subspace is expressed as:

$$\dot{\mathbf{z}} = \mathbf{f}(\mathbf{z}) \tag{5}$$

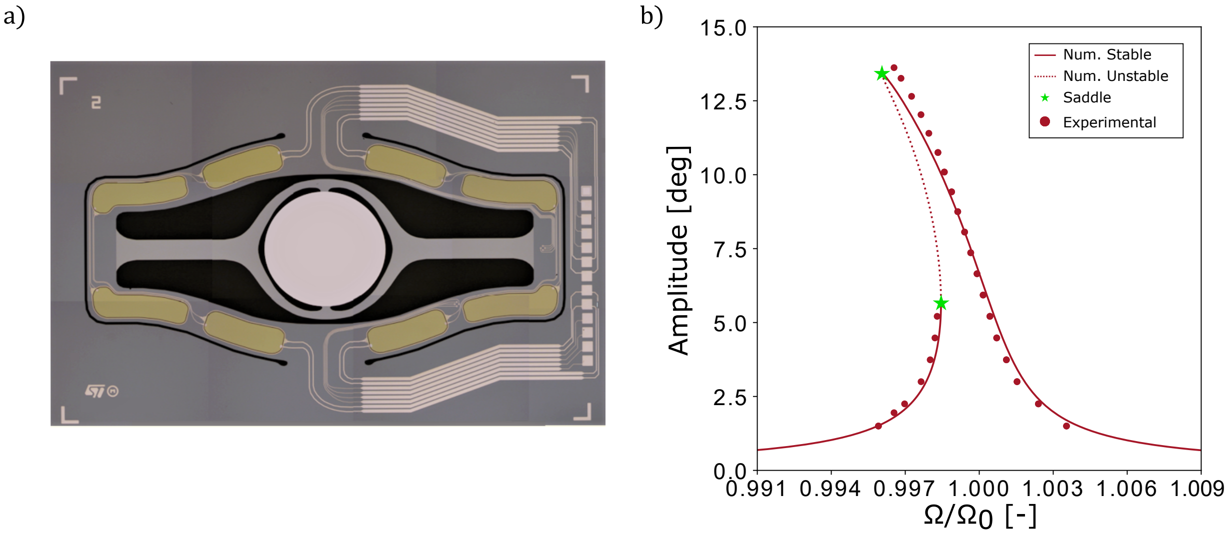

Figure 1: (a) optical microscope image of the tested device. (b) Preliminary comparison between experimental and numerical frequency response curves. Numerical curves are obtained using a complex normal form style parametrisation.

with $\mathbf{f}(\mathbf{z})$ reduced dynamics. An important remark of the presented formulation is that the polarization field $\mathbf{p}$ is assumed as known. In the present work, the polarisation is measured from experimental data and fed to the model to compute the piezoelectric forces $\mathbf{F}^p(\mathbf{U}, \mathbf{p})$ beforehand. This provides an efficient modeling approach as already evidenced in past works. When combined with the proposed model order reduction strategy it provides an efficient framework for predicting the nonlinear dynamic response of piezoelectrically actuated microstructures.

First results reported on a MEMS device are reported in Fig. 1. Fig. 1(a) reports the device under consideration, which is a MEMS micromirror developed by STMicroelectronics. In Fig. 1(b) a preliminary comparison between numerical and experimental frequency response curves is reported, highlighting the remarkable agreement of the proposed approach.

Conclusion

We here reported a model order reduction method for nonlinear piezoelectric structures based on the direct parametrisation for invariant manifolds. The starting equation is the conservation of linear momentum for structures subjected to finite transformations and to converse piezoelectric effect, the latter modeled using the Landau-Devonshire theory of ferroelectrics. The resulting system exhibits a follower force controlled by the polarization of the piezoelectric material and by the displacement of the structure, which in turn modifies the vibratory response of the system. Preliminary results obtained on a real MEMS device show promising results both in terms of computational performance and accuracy.

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