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► **To cite this version:**

Adeline Kabacinski, Eduardo Oliva, Fabien Tissandier, Julien Gautier, Michaela Kozlová, et al..
Spatio-temporal couplings for controlling group velocity in longitudinally pumped seeded soft X-ray
lasers. *Nature Photonics*, 2023, 17 (4), pp.354-359. 10.1038/s41566-023-01165-5 . hal-04107648

HAL Id: hal-04107648

<https://ensta-paris.hal.science/hal-04107648>

Submitted on 17 Nov 2023

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2. Supplementary Information:

Item	Present?	Filename Whole original file name including extension. i.e.: Smith_SI.pdf. The extension must be .pdf	A brief, numerical description of file contents. i.e.: <i>Supplementary Figures 1-4, Supplementary Discussion, and Supplementary Tables 1-4.</i>
Supplementary Information	Yes	Supplementary_information_NPHOT_2022_07_00928A.pdf	Supplementary discussion : <ul style="list-style-type: none">- Dephasing length and need for plasma dispersion compensation in the case of seeded SXRLs (figure 1 and 2, + 1 reference)- Energy measurements and definition of energy extraction (figure 3)
Reporting Summary	No		

4

5 Spatio-temporal couplings for controlling group velocity in longitudinally
6 pumped seeded soft X-ray lasers

7
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19
20 **Abstract:** Controlling the group velocity of an ultrashort laser pulse by spatio-temporal
21 couplings has been proposed to overcome inherent limitations in laser-plasma interactions.
22 We here report how this method improves the performance of a seeded soft X-ray laser
23 (SXRL), which intrinsically suffers from the group velocity mismatch between the infrared pump
24 beam, used to generate the plasma amplifier, and the soft X-ray seed beam. The energy
25 extraction was measured to be 3 times higher when the pump group velocity varies from 0.55c
26 to 1.05c. Moreover we demonstrate that the SXRL pulse duration is governed by the pump
27 beam velocity. By compensating the natural group velocity mismatch, the SXRL pulse duration
28 can be kept constant along its propagation, resulting in energetic pulses (up to 2 μ J) as short
29 as 350 fs. Such achievements constitute the first experimental demonstration of the so-called
30 "flying focus effect" to an application by controlling the group velocity of a high intensity laser
31 pulse propagating in a plasma.

32
33 Laser-plasma interaction arouses great interest as it abounds in applications in scientific,
34 medical and industrial domains^{1,2}. However, a plasma is a negative dispersion medium in
35 which the propagation velocity of a wave depends on its wavelength: the longer the
36 wavelength, the slower the wave propagates. This effect becomes more and more significant
37 as the plasma density increases, and often has limiting consequences. For instance, in laser-
38 wakefield accelerators³, dephasing originates from the difference between the laser and the
39 electron bunch velocities. This causes electrons to move into a phase region of the plasma
40 that is decelerating, therefore restricting the acceleration length and the electron beam energy
41 gain. In photon accelerators⁴, high plasma densities are required as they produce a strong
42 refractive index gradient and consequently a strong associated frequency shift. Nevertheless,
43 these favorable conditions are accompanied by stronger plasma dispersion resulting in faster
44 decoupling between the photon beam and the ionization wave, and thus in shorter interaction
45 lengths. In longitudinally pumped seeded SXRLs, the plasma dispersion results in a
46 desynchronization between the infrared beam used to pump the amplifier and the soft X-ray
47 seed beam. It inevitably leads to a temporal stretching of the SXRL pulse and to a poor energy
48 extraction from the energy contained in the ASE to the amplified seed. Moreover, high electron
49 densities and long amplifiers are the key for SXRLs optimization, but in these conditions the
50 natural desynchronization between the gain and the seed gets so strong that it prevents
51 amplification (see Supplementary information Figure 1 and Figure 2). For further improvements
52 of the source, compensating the plasma dispersion will therefore become the most limiting
53 factor and will have to be systematically addressed.

54
55 A hope to overcome these bottlenecks is to control the group velocity of the laser beam along
56 the Rayleigh length. Several techniques have been proposed for this purpose^{5,6,7,8}. One of
57 them is based on the use of two spatio-temporal couplings (STCs). Pulse Front Curvature
58 (PFC, quantified by α [fs/cm²] equivalent in the far field to longitudinal chromatism) induces the

59 different spectral components of a beam to be focused at different longitudinal positions.
60 Coupled with temporal chirp (quantified by β [fs²]) which changes the timing between these
61 different wavelengths, PFC enables to control the velocity of the peak intensity of an ultrashort
62 pulse on axis⁹. This phenomenon is known as the "flying focus effect". Depending on the
63 amount of those STCs, the peak intensity can be deposited either from the back to the front of
64 the focal region, or in the opposite direction. Subluminal to superluminal pulses have therefore
65 been demonstrated¹⁰ paving the way for novel control of laser-plasma interactions. With this
66 method, the ability to adjust the velocity of a femtosecond pulse in vacuum has been
67 demonstrated¹¹, as well as the ability to drive ionization waves of arbitrary velocity which has
68 been experimentally observed with moderate laser intensities^{12,13}. Although simulations
69 promise breakthroughs for photon acceleration¹⁴ or Raman amplification¹⁵, direct experimental
70 application with high intensity lasers has not been reported yet to date.

71
72 The present work applies this "flying focus effect" to a high intensity laser pulse (4×10^{18}
73 W/cm²) propagating in a plasma and demonstrates the benefit of controlling group velocity in
74 laser-plasma interaction in the benchmark case of a longitudinally pumped seeded SXRL. The
75 development of SXRLs has been motivated by the hope to carry out, at laboratory scale, single-
76 shot dynamical studies in the nanometric wavelength domain. Seeded SXRLs turn out to be
77 energetic sources¹⁶ available in a wide spectral range^{17,18,19}. Their amplifying medium is a
78 highly charged plasma, created by the interaction of an intense laser beam with a solid²⁰ or a
79 gas²¹. The population inversion is achieved thanks to collisional excitation, resulting in a strong
80 amplification of spontaneous emission (ASE) in the soft X-ray range (typically 3,5 μ J). The
81 spatial properties of the SXRL beam can be improved by seeding the amplifier with high-order
82 harmonics (HH)^{22,23} resulting in a coherent gaussian-like diffraction-limited beam²⁴. The
83 harmonics and the seeded SXRL energies being respectively below 1 nJ and close to 2 μ J,
84 the amplification factor is around 2000.

85
86 Nevertheless, seeding process has intrinsic limitations as the soft X-ray seed naturally
87 propagates faster than the infrared pump into the plasma amplifier. This group velocity
88 mismatch increases at high electron densities, which are required to generate short gain life
89 time as well as intense amplification^{25,26}. For instance, at an electron density $n_e = 10^{20}$ cm⁻³,
90 the soft X-ray beam basically propagates at c , whereas the infrared pump beam only
91 propagates at $0.97c$. It consequently conducts to a desynchronization between the arrival of
92 the seed and the sub-ps gain window²⁵ preventing from centimeter scale amplification length.
93 For transient SXRLs, the use of a traveling wave has been proposed to match the heating of
94 the plasma with the propagation of the soft X-rays photons in the plasma column²⁷. However,
95 this technique is not suitable for longitudinal geometry and has never been used to compensate
96 the plasma dispersion.

97
98 In this work, we investigate how the desynchronization between the gain and the seed affects
99 the energy and pulse duration of SXRL by harnessing spatio-temporal couplings (PFC and
100 temporal chirp) to control the group velocity of the pump beam. The amount of PFC is varied
101 by inserting chromatic doublets of infinite focal length in the pump beam, and the amount of
102 temporal chirp is adjusted with the distance between the gratings of the pump compressor (see
103 figure 1, see Methods). PFC enables to deposit the intensity of the pump beam in a line along
104 the longitudinal direction. Temporal chirp changes the delay at which a given spectral
105 component arrives at a given longitudinal position. The group velocity of the pump beam can
106 therefore be set at a value different than the natural group velocity induced by plasma
107 dispersion at a given electron density.

108
109 First, we investigated the amount of energy contained in the amplified seed, and its proportion
110 with respect to the energy obtained in purely ASE regime (i.e. the maximum energy that can
111 be extracted) while varying the synchronization between the seed and the gain. This
112 measurement has been performed respectively in figure 2 (a) for $\alpha = 4$ fs/cm², (b) for $\alpha = -2$
113 fs/cm² and (c) for $\alpha = -9$ fs/cm². The uncertainty on α was ± 1 fs/cm². The pump beam group

114 velocity was controlled, for a given α which should be different from 0 to permit the adjustment,
115 with the amount of temporal chirp. The variation of the seeded soft X-ray laser signal versus β
116 is shown in blue in figure 2. As $|\beta|$ is increased, the SXRL signal naturally decreases due to
117 the drop of the pump intensity induced by the pulse stretching²¹. Without group velocity
118 adjustment, this decrease is supposed to be symmetrical with respect to the optimum chirp.
119 However, an asymmetric behaviour is observed: in (a), the signal sharply drops for positive
120 values of chirp and diminishes rather smoothly for negative chirps. In (c), this asymmetry is
121 reversed with a sharp drop for negative chirps and a smoother decrease for positive chirps. In
122 (b) the signal seems almost symmetrical, the coefficient α being close to 0.

123
124 On the same graph, the group velocity with respect to temporal chirp is plotted in red. The error
125 band corresponds to an uncertainty of ± 1 fs/cm² on α and 500 fs² (corresponding to a
126 systematic error of 100 μ m on the optimum compression) on β . The shape of the group velocity
127 with respect to β depends on α . In particular, the group velocity displays an asymptote at chirp
128 $\beta = \frac{2f^2\alpha}{\omega_0}$ (see Methods). The location of this asymptote therefore depends on α and explains
129 well the asymmetric behaviour observed on the seeded signal. Indeed, the desynchronization
130 between the seed and the gain region gets stronger and faster as the mismatch between the
131 soft X-rays and the infrared is increased. When the infrared propagates at c , the seed stays
132 synchronized with the gain optimum along the propagation. For small mismatches, the seed is
133 desynchronized with the gain optimum but keeps being amplified in different gain regions.
134 When the mismatch increases, the seed meets weaker gain regions during shorter times and
135 is therefore less and less amplified. The side of the asymptote in the group velocity is a critical
136 region where the signal drops sharply due to a significant desynchronization. On the contrary,
137 the signal declines more smoothly when the group velocity gently goes away from c .

138
139 The energy extraction is also shown (in green) in figure 2. It is calculated as the ratio of the
140 energy contained in the amplified seed to the energy measured in the ASE (without seeding,
141 see Methods and Supplementary information Figure 3). In (a) the energy extraction increases
142 from $19 \pm 6\%$ to $59 \pm 4\%$ between $0.55c$ and $1.05c$ obtained respectively for around -30000
143 fs² and a suited chirp of 2000 fs². We measure an optimum energy extraction of $58 \pm 16\%$ in
144 (b) and $37 \pm 6\%$ in (c), obtained for the investigated group velocities which are the closest to
145 c in each case. The energy extraction is therefore optimal when compensating the natural
146 dispersion of the plasma, the seed being synchronized with the optimum gain, and is degraded
147 when the group velocity mismatch increases. It can be noted that the experimental conditions
148 were stable during a given scan at fixed α . However, due to difficulties to maintain identical
149 experimental conditions on a day-to-day basis (laser energy and beams overlapping), the
150 optimum energy extraction was not constant for the investigated configurations. We emphasize
151 that the dispersion compensation does not necessarily need a huge sacrifice in pump intensity
152 : for instance in (a), the group velocity is equal to c for $\beta = 300$ fs² which corresponds to a pulse
153 duration of 37 fs (instead of 30 fs at optimum chirp). It results in an intensity of 3×10^{18} W/cm²
154 (instead of 4×10^{18} W/cm²), sufficient to generate the lasing ions.

155
156 In a second time, we studied how the SXRL pulse duration depends on the infrared pump
157 beam group velocity. Pulse duration measurements have been performed at an electron
158 density close to $n_e = 7 \times 10^{19}$ cm⁻³, using a recently developed technique²⁶. Figure 3 shows
159 with blue symbols the measured SXRL RMS pulse duration with respect to the gas jet length
160 for different group velocities. Linear regressions of the pulse duration measurements with
161 respect to the jet length are plotted in red. The graph shows different behaviours depending
162 on the group velocity mismatch. For a strong desynchronization (in (a), $v_g = 1.09c \pm 0.04c$), the
163 pulse duration increases quickly with respect to the jet length with a slope of (89 ± 10) fs/mm.
164 For a minor desynchronization (in (c), $v_g = 0.98c \pm 0.02c$), pulse duration increases more
165 gently. Measurements have been performed for two different electron densities for this group
166 velocity ($n_e = 6 \times 10^{19}$ cm⁻³ and $n_e = 8 \times 10^{19}$ cm⁻³). The difference in their respective slopes
167 can be explained by the uncertainty on the group velocity, here comprised between $0.96c$ and

168 c for both cases but not necessarily identical. In the case (b) where $v_g = 1.00c \pm 0.03c$, the
169 pulse duration remains nearly constant with respect to the jet length, with a slope of (6 ± 10)
170 fs/mm. Experimentally, the stronger the group velocity mismatch, the more the SXRL pulse is
171 stretched when propagating along the amplifier.
172

173 This behaviour has been investigated using the DAGON 3D Maxwell-Bloch code²⁸ for an
174 electron density $n_e = 8 \times 10^{19} \text{ cm}^{-3}$. The simulated amplified pulse intensity (false color) and
175 population inversion (grayscale) for different group velocities and after different propagation
176 lengths are shown in figure 4. When the group velocity is equal to c (upper row), the SXRL
177 pulse duration remains constant along the propagation in the plasma as the seed beam is
178 synchronized with the gain region. On the contrary, when the group velocity is different from c
179 (lower row, here $0.95c$), the soft X-rays propagate faster than the infrared. This
180 desynchronization results in a stretching of the SXRL during its propagation as the tail of the
181 soft X-ray pulse is progressively amplified.
182

183 In figure 5 (a), the RMS simulated pulse duration is plotted with respect to the jet length and
184 for a group velocity ranging from $0.95c$ to $1.1c$. The SXRL pulse duration is almost constant to
185 around 350 fs at c , is increasing from 350 fs to 900 fs between 1 and 10 mm at $1.05c$, and
186 from 350 fs to 1.7 ps between 1 and 10 mm at $1.1c$. Therefore, at a given electron density and
187 a given plasma amplifier length, the pulse duration can be adjusted in a wide range thanks to
188 the control of the group velocity by STCs. The simulated RMS pulse duration with respect to
189 group velocity after 10 mm of propagation is shown in figure 5 (b). The pulse duration is clearly
190 reduced (from 1.7 ps at $1.1c$ to 350 fs at c) when locking the pump and seed velocities. It can
191 be noted that the simulated pulse duration is here 20 fs shorter at $0.99c$ than at c . This can
192 probably be explained by the RMS criterion as it takes into account the shape of the pulse
193 which depends on the group velocity.
194

195 Consequent increase of the SXRL seed amplification can be achieved resorting to high
196 electron densities and long amplifying media^{25,26} crucially requiring plasma dispersion
197 compensation (see Supplementary information Figure 2). Moreover, as demonstrated here,
198 the energy which can be gained by lengthening the amplifier is obtained at the expense of the
199 pulse duration if the dispersion is not compensated. This cost gets increasingly important when
200 the electron density and the amplifier length are raised as the desynchronization gets stronger.
201 For a natural group velocity $v_g = 0.97c$ at an electron density close to 10^{20} cm^{-3} , simulations
202 predict a stretching from 350 fs at 1 mm up to 640 fs at 10 mm, i.e. a pulse almost twice longer
203 than it would be if the pump velocity was artificially set at c thus allowing to keep the pulse
204 duration constant along the propagation. Furthermore, as established previously, controlling
205 the pump group velocity also allows for an optimization of the energy extraction. Therefore,
206 dispersion compensation offers great potential for SXRLs optimization and fosters
207 opportunities for applications requiring high intensities.
208

209 Controlling the natural velocity mismatch between the pump and the seed beams also paves
210 the way for applications requiring pulses at the femtosecond time scale. Collisional SXRL pulse
211 duration has been ranging for decades between 1.1 ps²⁹ and 100 ps¹⁶. It has been
212 experimentally shown only recently that femtosecond durations are achievable by quenching
213 the amplification while raising the electron density of the plasma amplifier²⁶. Figure 3 (c)
214 illustrates that the pulse length can be significantly shortened by combining the pulse duration
215 reduction induced by the increase of electron density (here from 6 to $8 \times 10^{19} \text{ cm}^{-3}$) and by the
216 control of the pump group velocity (here set at $0.98c \pm 0.02c$). It ensues a measured RMS
217 duration of 350 ± 200 fs in agreement with the simulations, obtained with energetic pulses (up
218 to 2 μJ) generated with a 10 mm plasma amplifier, which is the shortest collisional SXRL pulse
219 duration ever reported.
220

221 In summary, the suppression of the group velocity mismatch improves the performance of
222 current state-of-the-art SXRLs, and offers a considerable potential to continue on this path.

223 The compensation of natural dispersion in SXRLs fosters opportunities for many applications
224 requiring ultrashort pulses, or high intensities such as non linear XUV optics³⁰, and provides
225 an avenue to perform single-shot high resolution dynamical studies at the femtosecond time
226 scale³¹ with a table-top source. This study also represents the first direct application of the
227 "flying focus effect" to a high intensity laser pulse propagating in a plasma, and illustrates that
228 fundamental limitations of laser-plasma interactions induced by plasma dispersion can be
229 overcome thanks to STCs, opening prospects in a wide scope of domains such as photon
230 acceleration or laser wakefield acceleration.

231
232 Acknowledgements : This work is supported by "Investissement d'Avenir" (ANR-10-LABX-
233 0039-PALM (F.T. and S.S.) and ANR-18-EURE-0014 (A.K.)) and has received funding from
234 the European Union's Horizon 2020 research and innovation program under grant agreement
235 No. 654148 Laserlab-Europe. This work is also supported by Grant agency of the Czech
236 Republic, project number 18-27340S (M.Z.). The authors acknowledge support from COST
237 (European Cooperation in Science and Technology) for funding the Action TUMIEE (CA17126)
238 supporting this work. This work is also supported by the Universidad Politécnica de Madrid and
239 the Comunidad Autónoma de Madrid, línea de actuación estímulo a la investigación de jóvenes
240 doctores, project CROM and the Spanish Ministerio de Ciencia e Innovación through a Ramón
241 y Cajal RYC2018-026238-I fellowship (E.O.), and by Plan Estatal de Investigación Científica,
242 Técnica y de Innovación (Spain Ministerio de Ciencia e Innovación), grant PID2021-
243 124129OB-I00 (E.O.).

244
245 Author Contributions Statement : A.K., F.T., J.G, M.K. and S.S. designed and built the
246 experiment, and performed the measurements. A.K. analysed the experimental results. E.O.
247 developed the DAGON code. E.O. and A.K. performed the numerical simulations. J.-P.G.
248 operated the upgraded laser system of 'Salle Jaune'. I.A., C.T. and P.Z. supported the project.
249 All the authors contributed to the writing of the paper.

250
251 Competing Interests Statement : The authors declare no competing interests.

252
253 Tables (empty)

254
255 Figure Legends :

256
257 Figure 1 (SXRL&vg.ppt) : **Experimental setup**. Generation of the seeded soft X-ray laser with
258 a longitudinal pumping scheme. The group velocity v_g^{pump} of the pump beam is controlled
259 thanks to the adjustment of spatio-temporal couplings. Pulse Front Curvature, characterized
260 by the parameter α , is varied by inserting chromatic doublets of infinite focal length in the beam
261 before compression. Temporal chirp, characterized by β , is adjusted by varying the distance
262 between the gratings of the compressor.

263
264 Figure 2 (Extraction.ppt) : **Measurements of the impact of the group velocity on the SXRL**
265 **energy**. In blue, seeded SXRL signal with respect to temporal chirp β . In green, energy
266 extraction as a ratio of energy contained in the seeded beam to the energy in ASE.
267 Measurement points in blue and green are presented as mean values ($N = 5$) +/- SEM. In red,
268 group velocity of the infrared with respect to temporal chirp. The measurements are performed
269 for three different values of α : respectively +4 fs/cm² in (a), -2 fs/cm² in (b), and -9 fs/cm² in
270 (c).

271
272 Figure 3 (Measurements.ppt) : **Measurements of the impact of the group velocity on the**
273 **SXRL pulse duration**. Evolution of the pulse duration with respect to the jet length for different
274 group velocities. In blue, measurements of the RMS pulse duration of the SXRL with respect
275 to the jet length. Measurements have been performed for different group velocities of the pump
276 beam (respectively larger, equal and smaller than c) at an electron density close to $n_e = 7 \times$
277 10^{19} cm^{-3} . In red, linear regressions of these measurements. Small symbols are used for

278 individual points, large symbols are used for averaging over few shots ($N = 5$). Data are
279 presented as mean values \pm SEM.
280

281 Figure 4 (Simulations3D.ppt) : **3D simulations of the amplified SXRL pulse.** Simulated
282 amplified pulse intensity (false color) and population inversion (grayscale) for a group velocity
283 of c (upper row), and $0.95c$ (lower row) after respectively 1 mm, 5 mm and 10 mm from left to
284 right.
285

286 Figure 5 (Simulations.ppt) : **Simulations of the impact of the group velocity on the SXRL**
287 **pulse duration.** Simulations of the RMS SXRL pulse duration at $n_e = 8 \times 10^{19} \text{ cm}^{-3}$ (a) with
288 respect to the jet length for different group velocities of the infrared, (b) with respect to the
289 infrared group velocity for a 10 mm long jet.
290

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353

354 Methods :

355 **Laser system**

356 The experiment was conducted at the Laboratoire d'Optique Appliquée with the 'Salle Jaune'
357 Ti:Sa laser system, which delivers 3 beams at 1 Hz around a central wavelength of 820 nm: a
358 pump beam (1.5 J, 30 fs), a waveguiding beam composed of two pulses -an "ignitor" pulse
359 (150 mJ, 30 fs) followed 900 ps later by a "heater" pulse (700 mJ, 600 ps)-, a HH driver beam
360 (30 mJ, 30 fs). A part of the pump beam (20 mJ, 30 fs) is used for pulse duration
361 measurements.
362

363 **SXRL**

364 The soft X-ray amplifier at 32.8 nm was generated by focusing the intense infrared pump beam
365 with a spherical mirror ($f = 700$ mm) into the krypton gas jet, resulting in an intensity of 4×10^{18}
366 W/cm² enabling the generation of the lasing ion. A population inversion is then achieved thanks
367 to an efficient pumping by hot electrons, allowing for a lasing of the $3d^9 4d_{J=0} \rightarrow 3d^9 4p_{J=1}$
368 transition of nickel-like krypton at 32.8 nm²¹. The gas jet length can be adjusted under vacuum
369 between 4.5 and 10 mm thanks to a motorized blade covering a part of the slit nozzle.
370

371 To overcome the strong refraction of the pump beam induced by the plasma, an optically
372 preformed plasma waveguide was implemented using the second beam composed of two
373 pulses ("ignitor"+"heater"), which is focused by an axicon lens over the whole gas length³².
374 The electron density builds up as electrons generated by the "ignitor" get heated and foster
375 avalanche collisional ionization. The natural hydrodynamic plasma expansion then starts and
376 the generated electrons are ejected out of the optical axis, therefore allowing pump beam
377 propagation³³. The waveguide transmission was assessed around 40% by integrating the
378 whole energy of the beam profile imaged at the entrance and at the output of the plasma
379 channel. The electron density of the plasma amplifier was measured with a Mach-Zender
380 interferometer.
381

382 The soft X-ray seed^{22,23} is implemented by focusing an infrared driver beam into a gas cell filled
383 with argon. The on-target intensity was assessed at about 2×10^{14} W/cm². The 25th harmonic
384 signal was optimized by changing the gas pressure and the beam energy, tuning the beam
385 aperture with a motorized iris, adjusting the longitudinal position of the lens, and spectrally
386 tuned onto the amplifier ASE laser line at 32.8 nm by chirping the HH driver beam. The seeding
387 was fulfilled by imaging the harmonic output at the entrance of the plasma amplifier, using a
388 5° grazing-incidence gold-coated toroidal mirror in a 1.5–1 m conjugation. Special attention
389 has been paid to spatial and temporal matching. The spatial coupling of the HH beam and the
390 SXRL gain region was done in the infrared by overlapping the infrared driver with the focal spot
391 of the pump. The temporal overlap between the HH signal and the SXRL amplification period
392 was approached by synchronizing the infrared driver and the pump beams with an accuracy
393 of 30 fs.
394

395 **Adjustment of the pump beam group velocity**

396 The group velocity of the pump beam was controlled by adjusting the amount of two spatio-
397 temporal couplings: the Pulse Front Curvature (PFC) and the temporal chirp. PFC is equivalent
398 to longitudinal chromatism in the far field, and is quantified by its strength α in fs/cm² given by:
399
400

$$401 \quad z(\omega) = 2cf^2\alpha \frac{\Delta\omega}{\omega}$$

403

404 where $z(\omega)$ is the longitudinal position of the focusing plane of the frequency ω , f is the focal
405 length (here $f' = 700$ mm for the spherical mirror focusing the pump beam), and $\Delta\omega$ the
406 frequency offset with respect to ω_0 the central frequency of the pump beam. To introduce in
407 practice a precise amount of PFC in the beam, chromatic doublets of infinite focal length are
408 inserted before compression¹¹. Here the quantity of PFC has been varied working either with
409 2 doublets ($\alpha = +4$ fs/cm²), either with a single doublet ($\alpha = -2$ fs/cm²), either without doublet ($\alpha =$
410 -9 fs/cm²). The coefficient α was measured with an uncertainty of ± 1 fs/cm². It can be noted
411 that the amount of PFC can also be adjusted dynamically by changing the longitudinal position
412 of a single chromatic doublet inserted in a divergent beam³⁴.

413

414 The second STC needed is the temporal chirp, given by the parameter $\beta = -\left(\frac{d^2\phi}{d^2\omega}\right)_{\omega_0}$, the
415 second order derivative of the spectral phase in fs². In practice, the amount of temporal chirp
416 of the pump beam was adjusted by changing the distance between the gratings of the
417 compressor, since the coefficient β is proportionnal to it.

418

419 The group velocity v_g can finally be derived from the following formula :

420

421

$$\frac{v_g}{v_g^{nat}} = \frac{1}{1 - \left(\frac{\omega_0}{2f^2}\right)\left(\frac{\beta}{\alpha}\right)}$$

422

423 v_g^{nat} being the natural group velocity given by $v_g^{nat} = c \sqrt{1 - \frac{n_e}{n_c^\lambda}}$, with n_e the electron density of
424 the plasma and n_c^λ the critical density at a given wavelength.

425

426 Energy measurements

427 The energy measurement was performed using a 25° off-axis soft X-ray parabola of focal
428 length 200 mm, a deflecting plane mirror at a 80° incidence angle and a soft X-ray camera of
429 resolution 1024 × 1024 pixels. The interferential multilayer coating of the parabola designed to
430 yield a high reflectivity at 32.8 nm was deposited at the Institut d'Optique, France. The system
431 images a plane at about 1.4 m from the soft X-ray plasma source with a magnification of 0.6.
432 The signal has been measured in purely ASE regime (without seeding), and within the
433 amplified seed (with seeding). The energy extraction has been derived as the ratio of the
434 energy contained in the amplified seed to the total energy contained in the ASE, which
435 represents the maximum energy that can be extracted (See supplementary information Figure
436 3 for more details). These measurements have been performed for three different α coefficients
437 and for a distance between the compression gratings varied in the range -7 to +7 mm around
438 the optimum compression. Error bars were determined by repeatability over few shots
439 (standard error of the mean).

440

441 Pulse duration measurements

442 The SXRL pulse duration was measured using a single-shot temporal profiler described in²⁶.
443 The measurement is based on a pump-probe technique allowing to measure the transient
444 reflectivity of a laser-damaged soft X-ray mirror using the 32.8 nm laser as a probe. A soft X-
445 ray multilayer is irradiated simultaneously with the SXRL beam and with an intense infrared
446 femtosecond pulse in a travelling wave geometry. The envelope of the temporal profile can be
447 retrieved with a resolution as short as 200 fs. Uncertainties have been determined taking into
448 account this resolution and the repeatability over few shots. Measurements have been
449 performed at electron densities close to 7×10^{19} cm⁻³, for different jet lengths and group
450 velocities of the pump beam.

451

452 3D Maxwell-Bloch simulations

453 Simulations have been performed using the DAGON 3D Maxwell-Bloch code^{28,35}. The group
454 velocity of the pump beam can be varied as an input parameter of this code, which models the
455 full spatio-temporal structure of the amplification of high-order harmonics along the pulse
456 propagation in a krypton plasma amplifier. The creation and evolution of the plasma waveguide
457 is modelled with the 2D radiative hydrodynamics code ARWEN^{36,37}. This code solves the
458 equations of compressible fluid-dynamics coupled to energy transfer processes: electronic
459 diffusion, radiative transfer and laser propagation and absorption. The resulting density profile
460 is fed to a 2D axisymmetric Particle-In-Cell code, FBPIC³⁸. This code solves Maxwell's
461 equations and the relativistic motion equations for the macroparticles that model the plasma.
462 The temporal dynamics of the lasing ion and population inversion is modelled with the
463 collisional-radiative code OFIKinRad³⁹. Non-equilibrium effects are taken into account by
464 solving a Fokker-Planck equation to compute the electron energy distribution function and,
465 afterwards, the cross sections. The output of FBIC and OFIKinRad is used by DAGON to
466 compute the temporal dynamics of the plasma amplifier. DAGON solves the set of 3D Maxwell-
467 Bloch equations in the paraxial and slowly varying envelope approximation, taking into account
468 the temporal dynamics of the amplifier medium. The three-dimensional aspect of the code is
469 required to take into account the waveguide and the inhomogeneities of the plasma. These
470 inhomogeneities mainly come from the overionization induced in regions where the intensity
471 reaches the threshold to produce higher charged ions due to focusing effects⁴⁰. In addition to
472 this, the code also takes into account the 3D stochastic structure of the Amplified Spontaneous
473 Emission. We modelled a 100 μm x 100 μm x 10 mm amplifier, using a mesh consisting in 100
474 x 100 x 2000 cells. The timestep was fixed to 10 fs.

475

476 Data availability : The main datasets supporting the findings of this study are provided within
477 the paper. Additional data are available from the corresponding authors upon reasonable
478 request.

479

480 Methods-only references :

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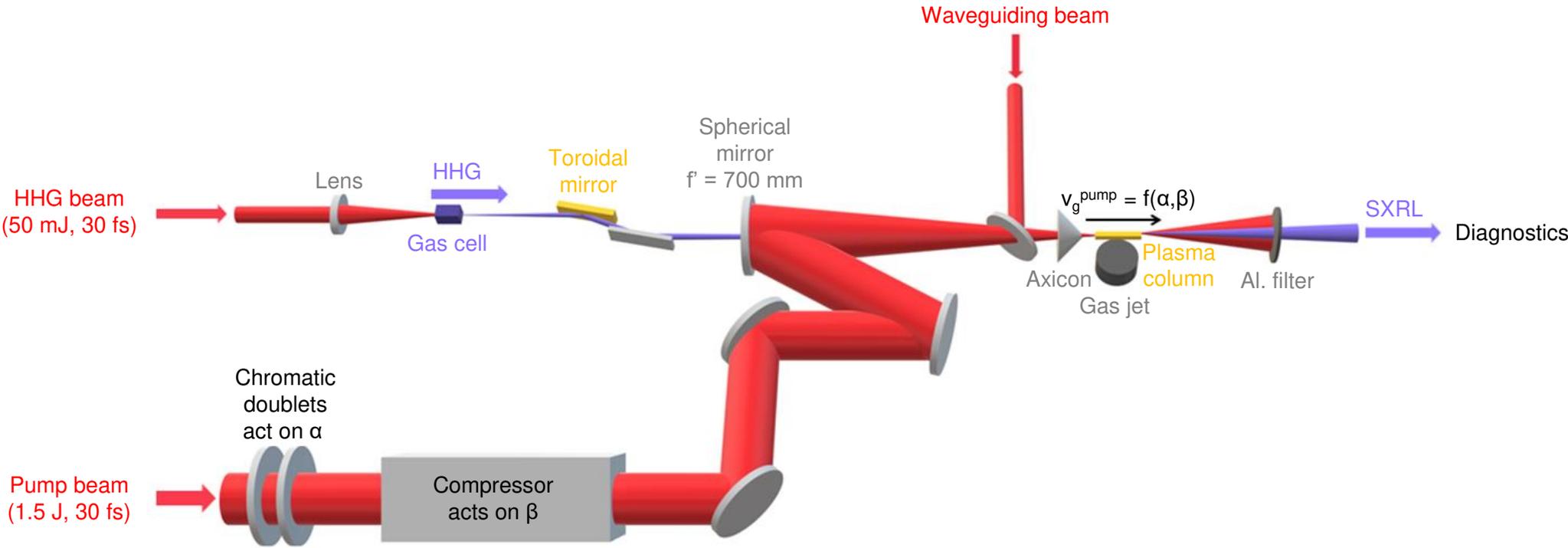
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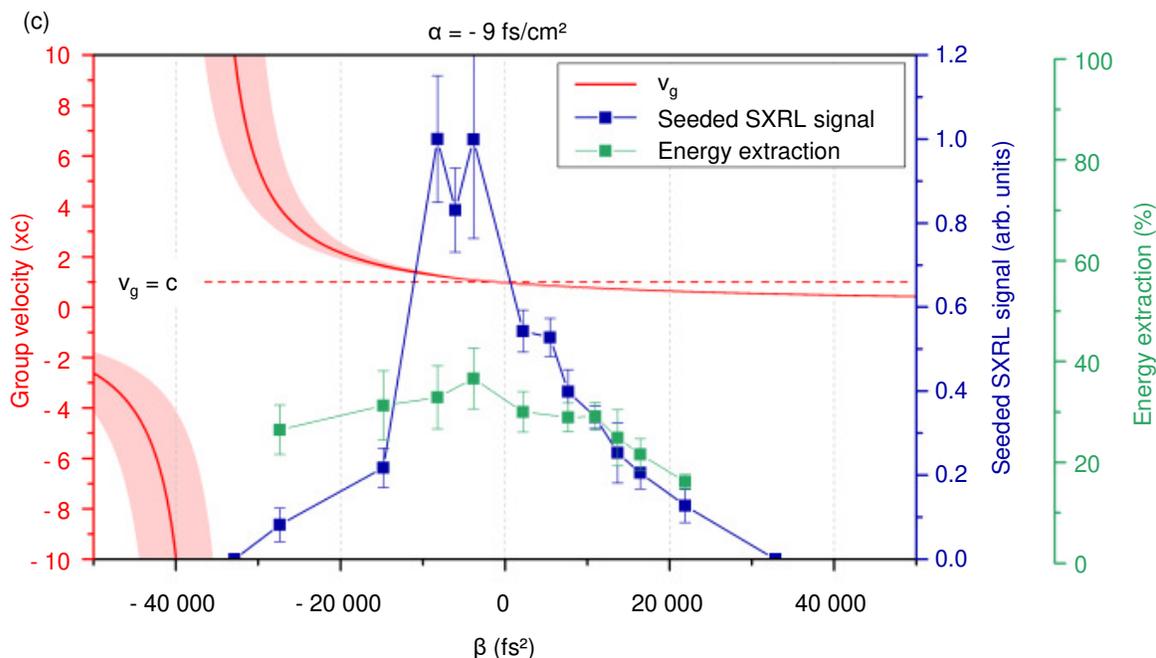
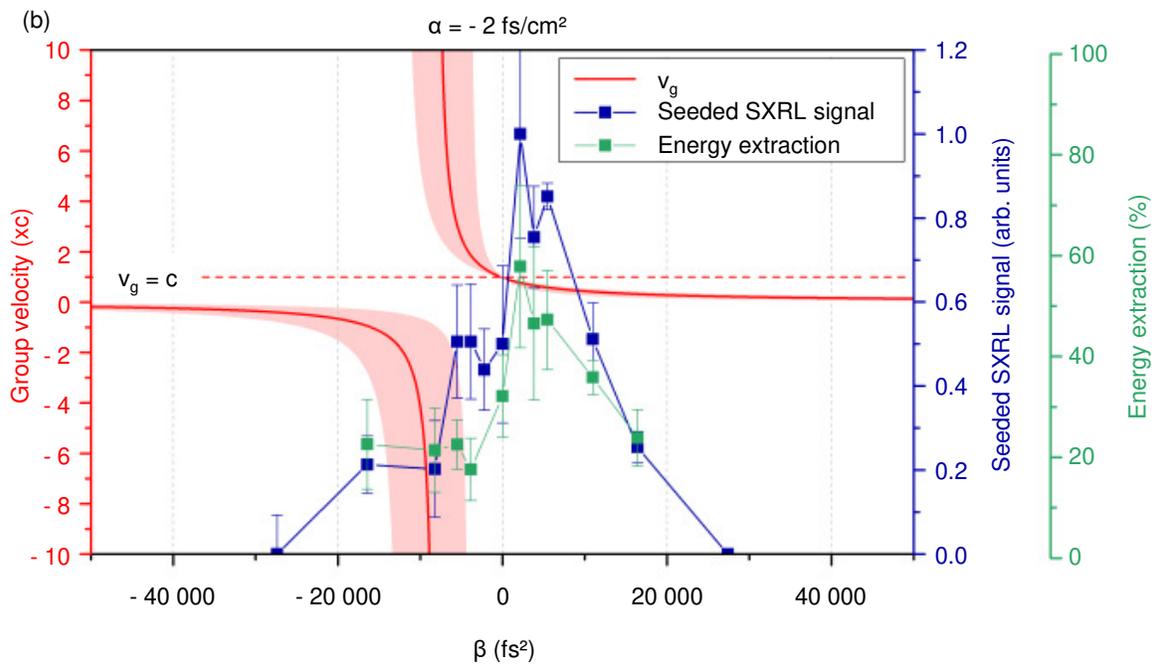
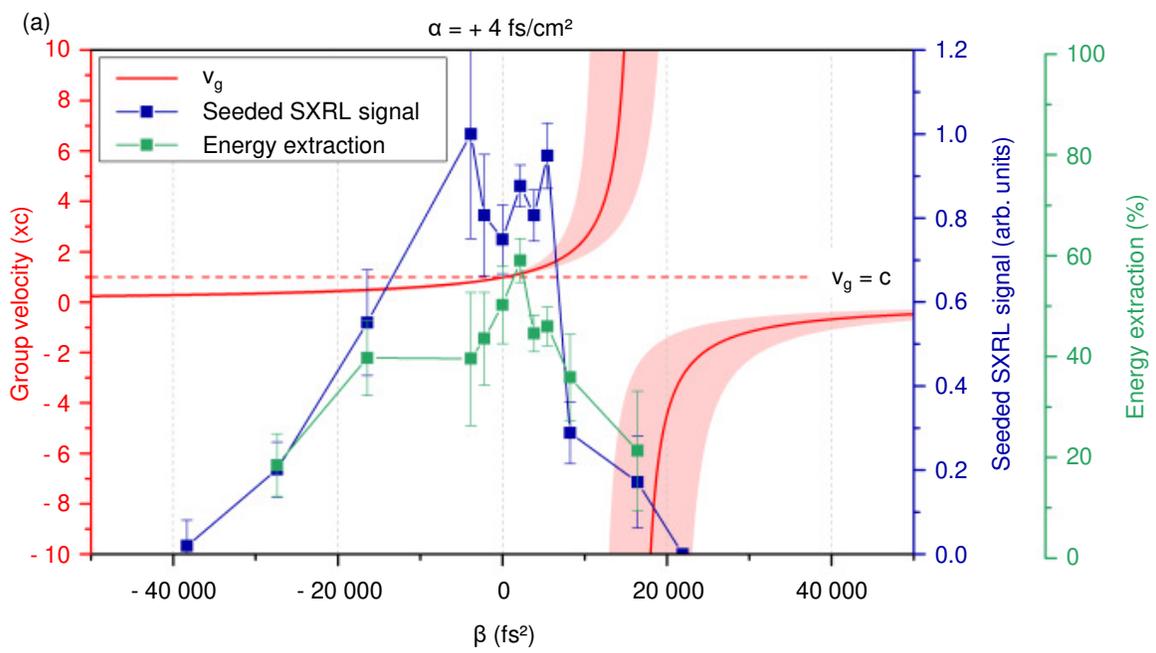
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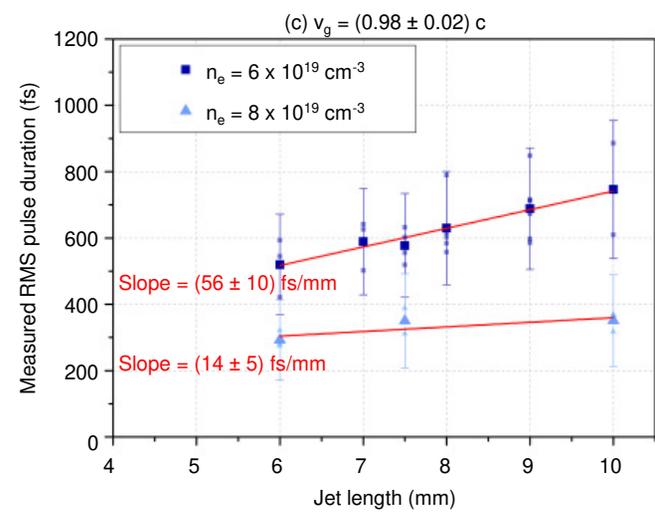
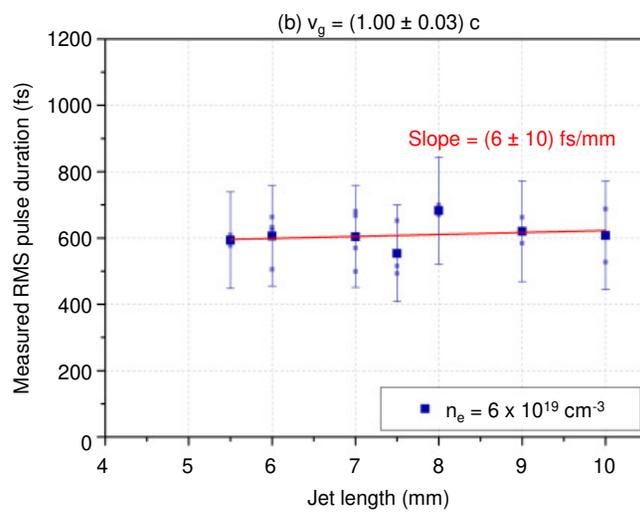
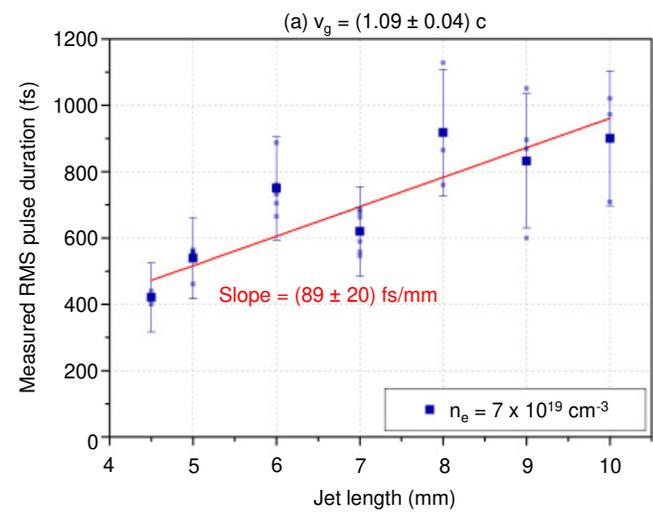
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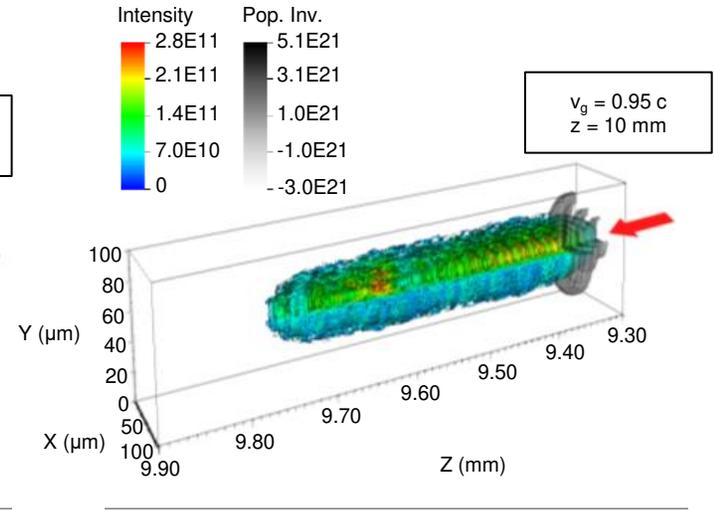
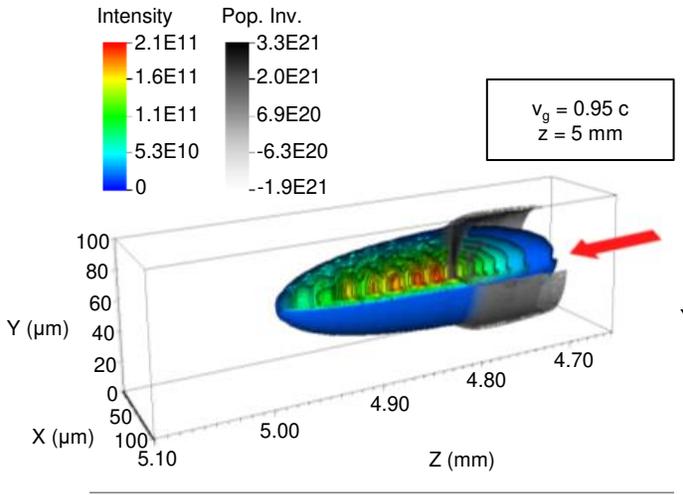
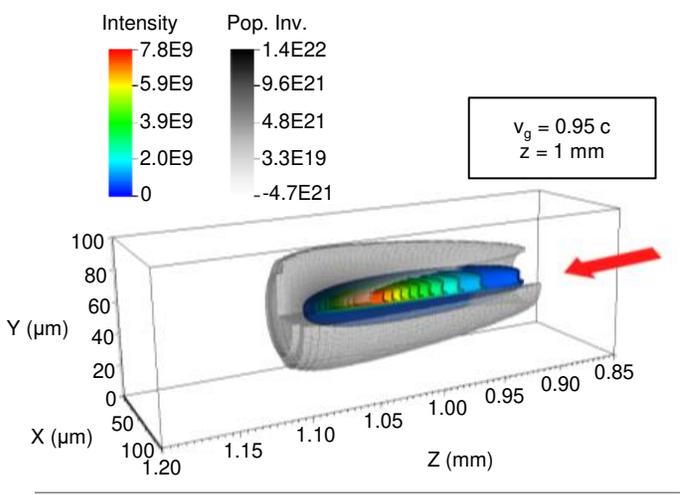
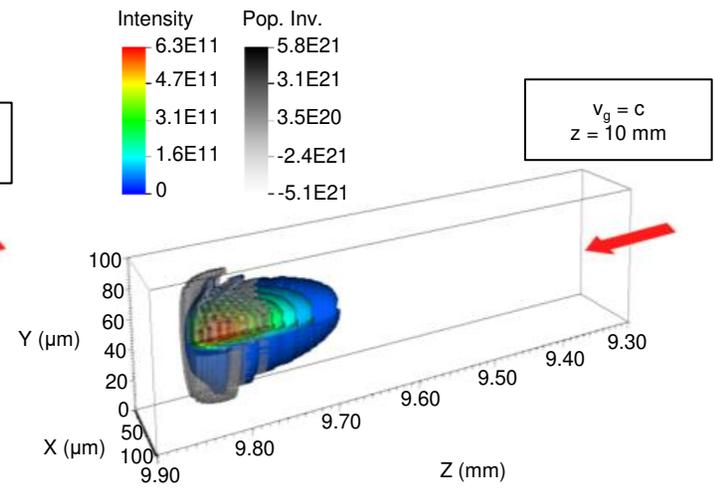
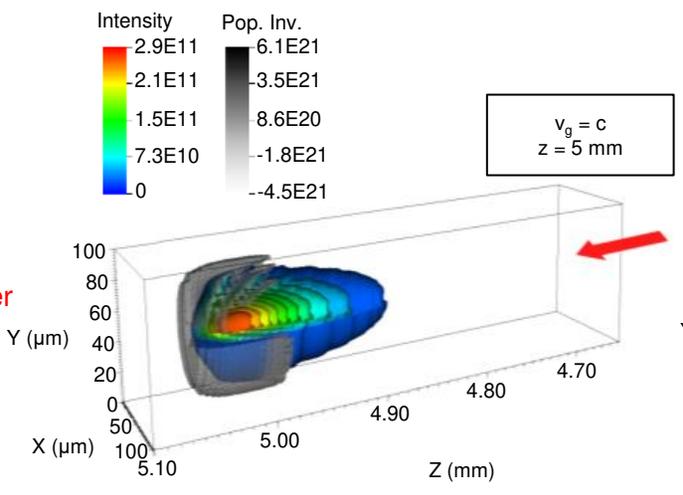
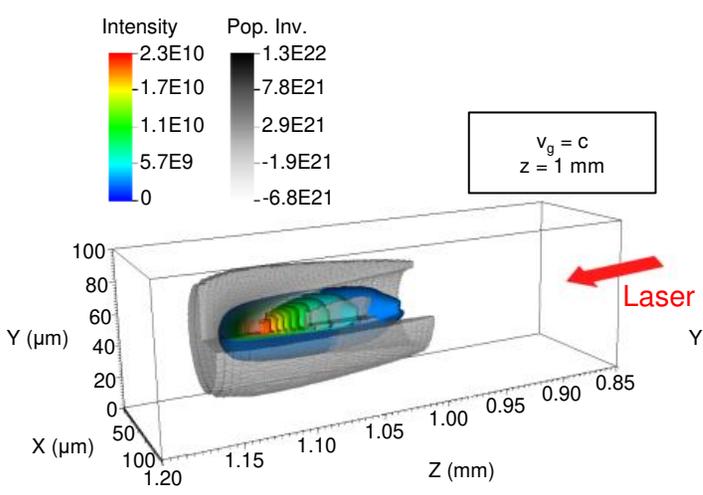
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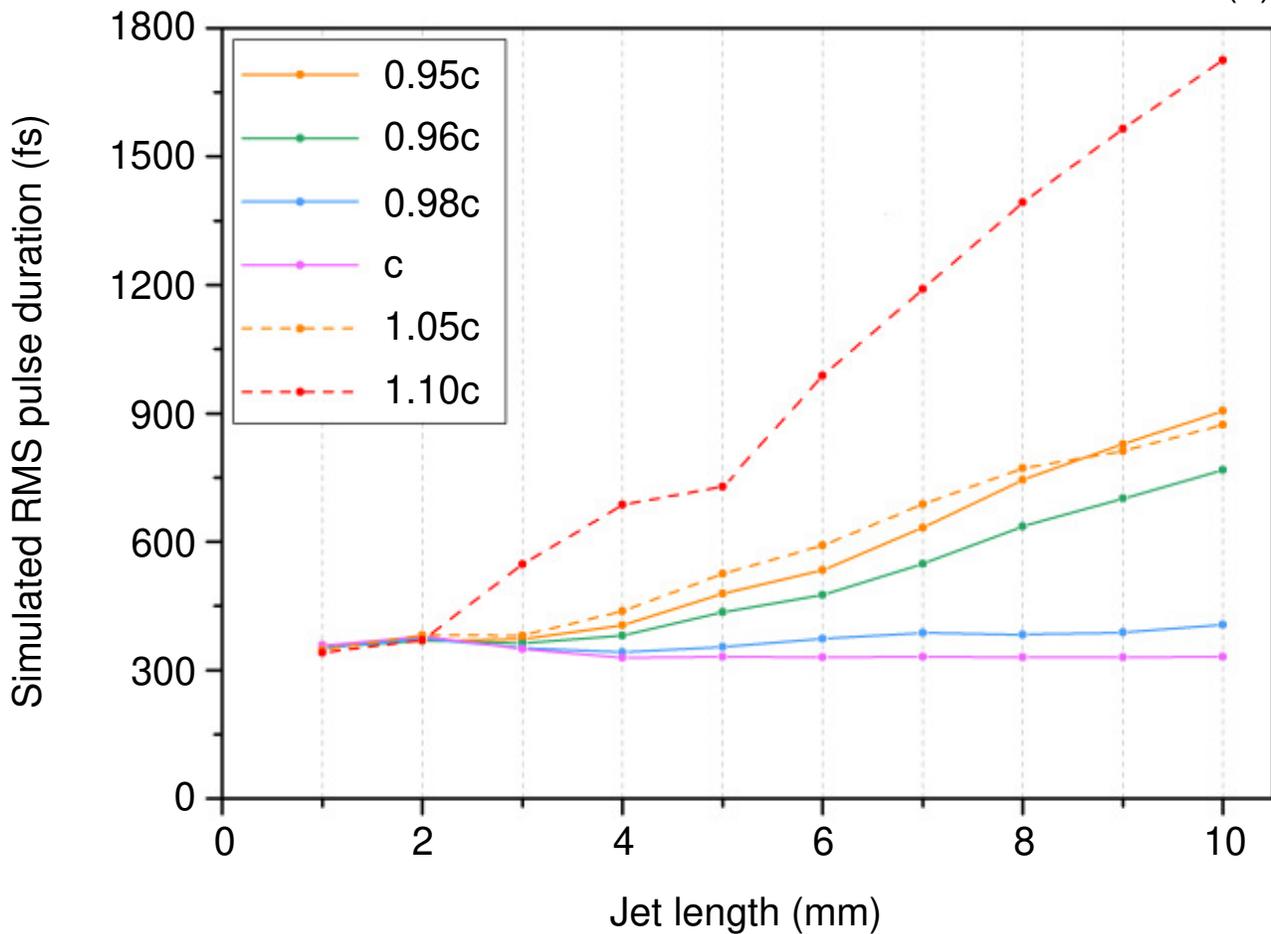








(a)



(b)

